FREQUENCY RESPONSE ANALYSIS AND DESIGN
OF
SINGLE-VALUED NONLINEAR SYSTEMS
USING THE PARAMETER PLANE

by

John Philip Davis

United States
Naval Postgraduate School

THESIS

FREQUENCY RESPONSE ANALYSIS AND DESIGN
OF
SINGLE-VALUED NONLINEAR SYSTEMS
USING THE PARAMETER PLANE

by

John Philip Davis

June 1969

Frequency Response Analysis and Design
of
Single-Valued Nonlinear Systems
Using the Parameter Plane

by

John Philip Davis
Lieutenant(junior grade), United States Navy
B.S., United States Naval Academy, 1968

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June, 1969

ABSTRACT

Frequency response techniques are a valuable tool in
the analysis and synthesis of linear systems. Extension
of these techniques is made to analyze and design systems
with a single-valued nonlinear element. The relationship
between the characteristics of a nonlinear element and the
frequency of the system is developed by simple calculations
and a digital computer program.

TABLE OF CONTENTS

3

LIST OF TABLES

5

LIST OF ILLUSTRATIONS

8

9

10

ACKNOWLEDGEMENTS

I. <u>INTRODUCTION</u>

Frequency response techniques are a valuable tool in the analysis and synthesis of linear systems. Reasons for this are, in part, the ease of presentation and manipulation of block diagrams and transfer functions, and in part, the ease of computation and interpretation of Bode diagrams and Nichol charts. It is therefore desirable to extend these techniques to include nonlinear systems. Block diagrams with nonlinear elements can be handled with the addition of some new rules. Handling transfer functions that include nonlinearities presents more of a problem. The nonlinear element's characteristics must be described in terms of frequency so that they can be treated mathematically with the linear part of the system. This thesis develops a technique which shows the relationship between the nonlinearity and the frequency response of the system. It then uses this relationship to perform analysis and synthesis of systems that contain nonlinearities.

Several assumptions are made and some conditions must be satisfied. The most important condition is that the nonlinearity be a single-valued nonlinearity. A single-valued nonlinearity is one that for any given input has only one output. This condition eliminates elements such as backlash, negative deficiency, two-position relay, relay with hysteresis, etc. By making this condition, a

13

single-valued nonlinearity can be expressed as a variable gain device, with the gain a function of the signal into the device.

Two other main conditions are required. The first is the condition that the input to the system be a pure sine wave of constant amplitude. The second condition is that the input signal to the nonlinear component is a pure sine wave.

In order to describe the nonlinearity as a function of frequency, it must first be described as a variable gain device. This gain is a function of the signal amplitude into the element. Using the PARAMS program, a plot of the gain versus the signal amplitude can be made. Each curve in the resulting family is for a fixed frequency. With this plot the gain can be described as a function of fre-quency. Using this relationship between gain and frequency, the analysis can be completed on a frequency response plot with constant gain curves.

The analysis and synthesis techniques use the PARAMS program (reproduced in the computer print-out section) to make two types of plots. The first type of plot is a plot of nonlinear gain versus the signal amplitude into the non-linear element with constant frequency. The second type of plot is a plot of the magnitude (or phase) of the open or closed loop function versus frequency. Each curve in the family is for a fixed value of gain for the nonlinear element.

A brief description of the operation of the PARAMS program is presented below. Let a transfer function be represented by,

$$T(s) = \frac{N(s)}{D(s)} \qquad (1.1)$$

Where $N(s)$ is of the form $A_o + A_1 s + A_2 s^2 + \cdots A_{n-1} s^{n-1} + A_n s^n$ and $D(s)$ is of the form $B_o + B_1 s + B_2 s^2 + \cdots B_{m-1} s^{m-1} + B_m s^m$.

Each A_k and B_k can be represented by,

$$A_k = C_k + D_k \alpha + E_k \beta + F_k \alpha\beta \qquad (1.2a)$$

$$B_k = G_k + H_k \alpha + I_k \beta + J_k \alpha\beta \qquad (1.2b)$$

Where α and β are two variable parameters of the system. $T(s)$ can be represented by,

$$T(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \qquad (1.3a)$$

and $T(-s)$ by,

$$T(-s) = \frac{N_e(s) - N_o(s)}{D_e(s) - D_o(s)} \qquad (1.3b)$$

Where $N_e(s)$ is the even order terms of the numerator of $T(s)$, $N_o(s)$ is the odd order terms of the numerator of $T(s)$, $D_e(s)$ is the even order terms of the denominator of $T(s)$, and $D_o(s)$ is the odd order terms of the denominator of $T(s)$.

The mangitude squared function is,

$$M^2 = [T(s)] [T(-s)] = \frac{[N_e(s)]^2 - [N_o(s)]^2}{[D_e(s)]^2 - [D_o(s)]^2} \qquad (1.4)$$

Each term of equation (1.4) is a even polynomial in s. Letting s = jw, equation (1.4) can be reduced to,[1]

$$M^2 = \frac{\sum\limits_{k=0}^{N} A_k \omega^{2k}}{\sum\limits_{k=0}^{m} B_k \omega^{2k}} \qquad (1.5)$$

Where A_k and B_k are given by equations (1.2a) and (1.2b).

By letting α or β be a constant, plots of the remaining variable parameter (β or α) versus magnitude can be made with frequency constant. In like manner, plots of magnitude versus frequency can be made with a constant parameter value.

Phase calculations can be made by utilizing the formula,

$$\Theta = \tan^{-1} \left[\frac{N_o(s) D_e(s) - N_e(s) D_o(s)}{N_e(s) D_e(s) - N_o(s) D_o(s)} \right] \qquad (1.6)$$

In this thesis α or β represents the variable nonlinear gain.

---

[1] Glavis, G.O., Frequency Response in the Parameter Plane, Master's Thesis, Naval Postgraduate School, 1967.

16

II. INTRODUCTION TO NONLINEAR FREQUENCY RESPONSE ANALYSIS USING THE PARAMETER PLANE

A. PREFACE

As an introduction to the technique of nonlinear frequency response analysis, a fourth order system will be considered. The analysis is divided into two parts. One part is the analysis of the system with a nonlinear element in the forward path. The second part is the analysis of the system with a nonlinear element in the major feedback path. Each part will be analyzed for a saturation element and then analyzed for a dead zone element. All the assumptions made in the first chapter are valid for this example and all other examples used.

The second section of this chapter looks closer at the jump resonance effect that a nonlinear element can cause. The example taken is a simple second order servo with a saturation element in the forward path. The analysis techniques are the same in this example as were those in the first example of this chapter.

B. ANALYSIS OF THE FIRST EXAMPLE

The first example of this analysis is a fourth order system with a nonlinear element in either the forward path or the main feedback path. The fourth order system chosen for this example is a simple position servomechanism which is shown in Figure (2.1).
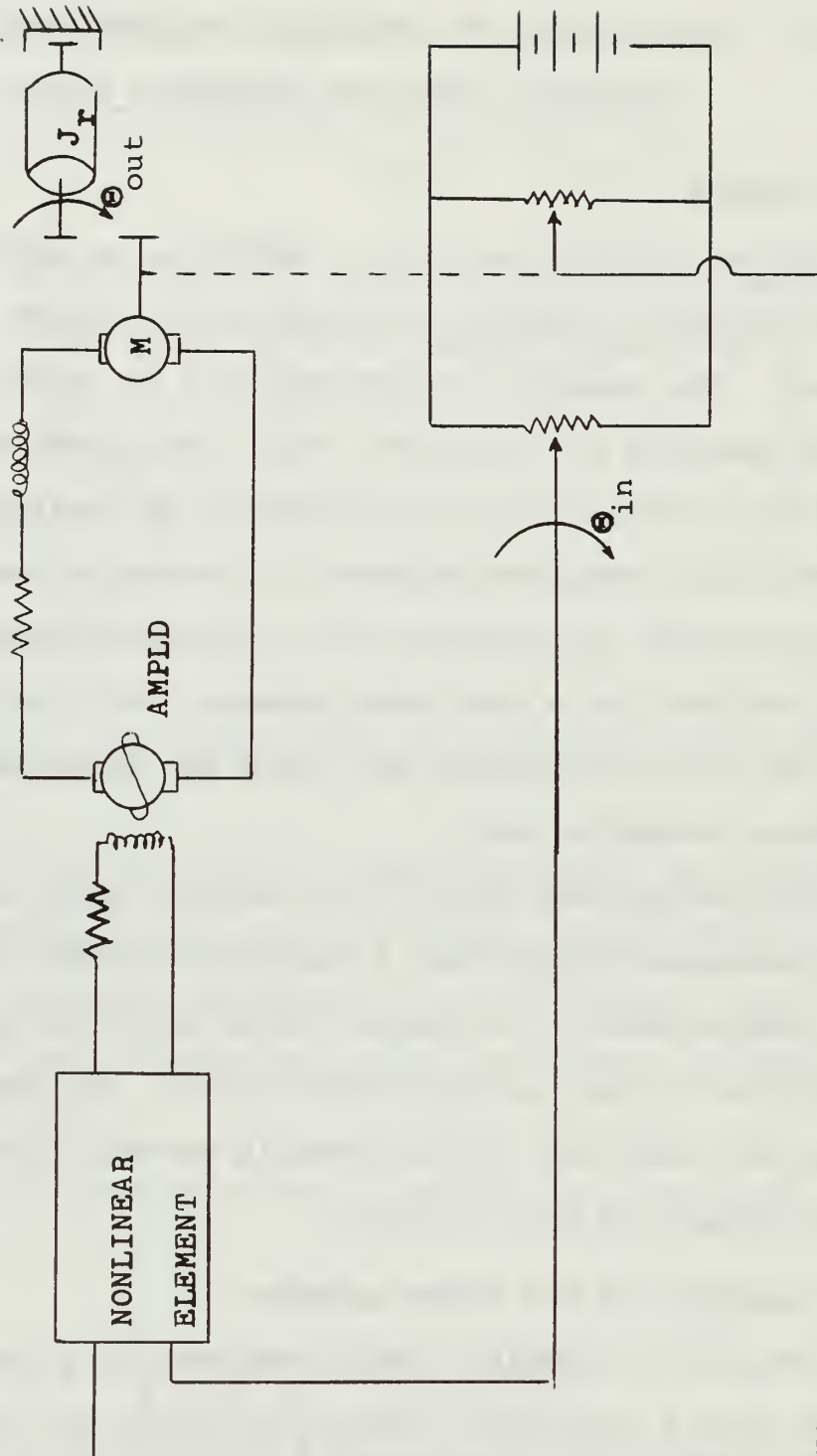
Figure 2.1   POSITION SERVOMECHANISM EXAMPLE OF CHAPTER 2

The transfer function of the amplidyne is,

$$G_a(s) = \frac{k_q}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_q})} \qquad (2.1)$$

where $\tau_f = \frac{l_f}{R_f}$ and $\tau_q = \frac{l_q}{R_q}$

The motor-load combination transfer function is,

$$G_m(s) = \frac{k_m}{s(s + \frac{1}{\tau_1})} \qquad (2.2)$$

Where τ_1 is the motor-load time constant and is pre-determined.

The error detector has a gain of k_e.

Putting the position servomechanism into block diagram form is done and is shown in Figure (2.2). The non-linear element in the forward path is fed by the error signal Θ_e. The output signal, Θ_a feeds the nonlinear element in the main feedback path. These nonlinear elements are represented by blocks with their associated variable gain. α is the variable gain for the nonlinear element in the forward path and β is the variable gain for the non-linear element in the main feedback path. The two nonline-ar elements are not in the system at the same time. Analysis will be done for the nonlinearity in the forward path and then for the nonlinearity in the main feedback path.

19
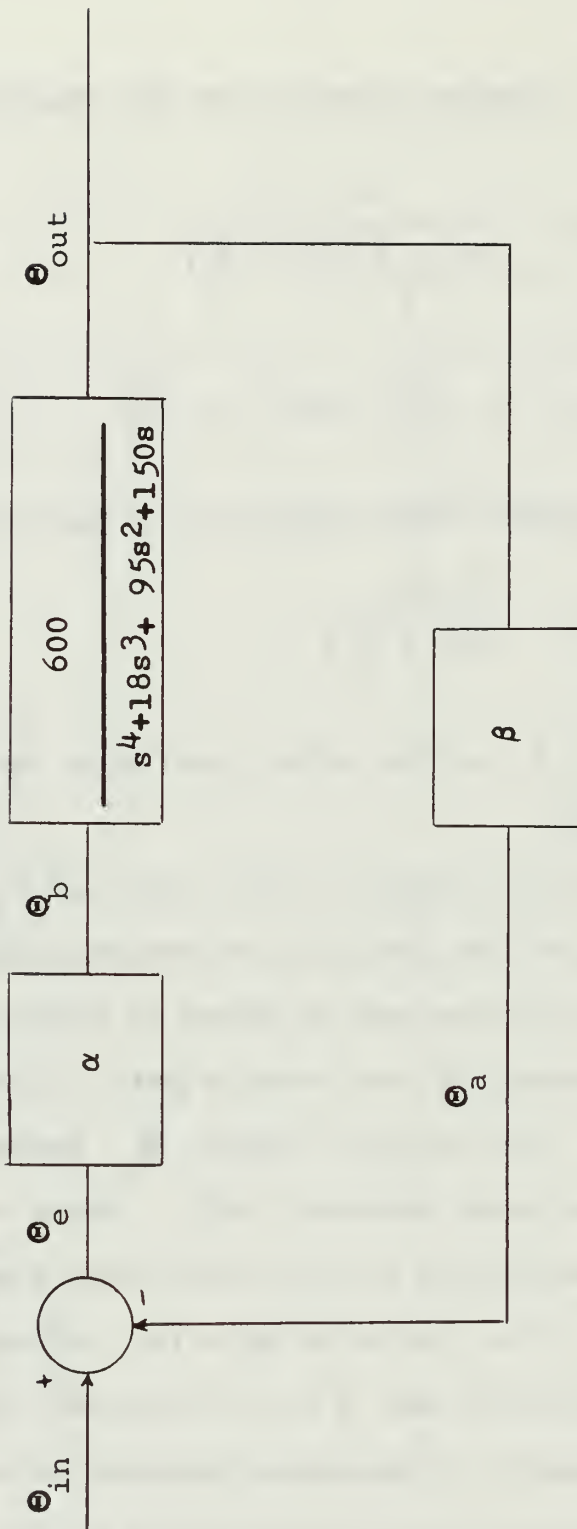
Figure 2.2 BLOCK DIAGRAM OF THE POSITION SERVOMECHANISM
OF CHAPTER 2

Assume that $\frac{1}{\tau_f} = 10.0$, $\frac{1}{\tau_q} = 5$, $\frac{1}{\tau_1} = 3$, and $k_q k_m k_e = 600$. The characteristic equation now becomes,

$$s^4 + 18s^3 + 95s^2 + 150s + 600\alpha\beta = 0.0 \qquad (2.3)$$

And the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha\beta} \qquad (2.4)$$

When analyzing the system with a nonlinearity in the forward path, β will be equal to unity. The converse is true when analyzing with the nonlinearity in the feedback path. α will then be equal to unity.

Many types of single-valued nonlinearities could be considered here. Two types are used in this example and in examples that will follow mainly for simplicity sake. These two types are saturation and dead zone. For this example the saturation element limits at 10.0. as shown in Figure (2.3a). Figure (2.3b) shows the dead zone element where the dead zone ends at 10.0. The assumed magnitude of Θ_{in} is 20.0. It could be any magnitude; however it must be specified for the analysis because the frequency response is a function of the magnitude of Θ_{in}. Because the gain of the nonlinear element is a function of the signal going into it, what is desired is a relationship between the gain of the nonlinear element and the input signal. However, it is hard to describe the signal going into the nonlinear element as a function of the nonlinear gain. It is easier to

21

Signal into the
element

10.0

Signal out of
the element

10.0

-10.0

-10.0
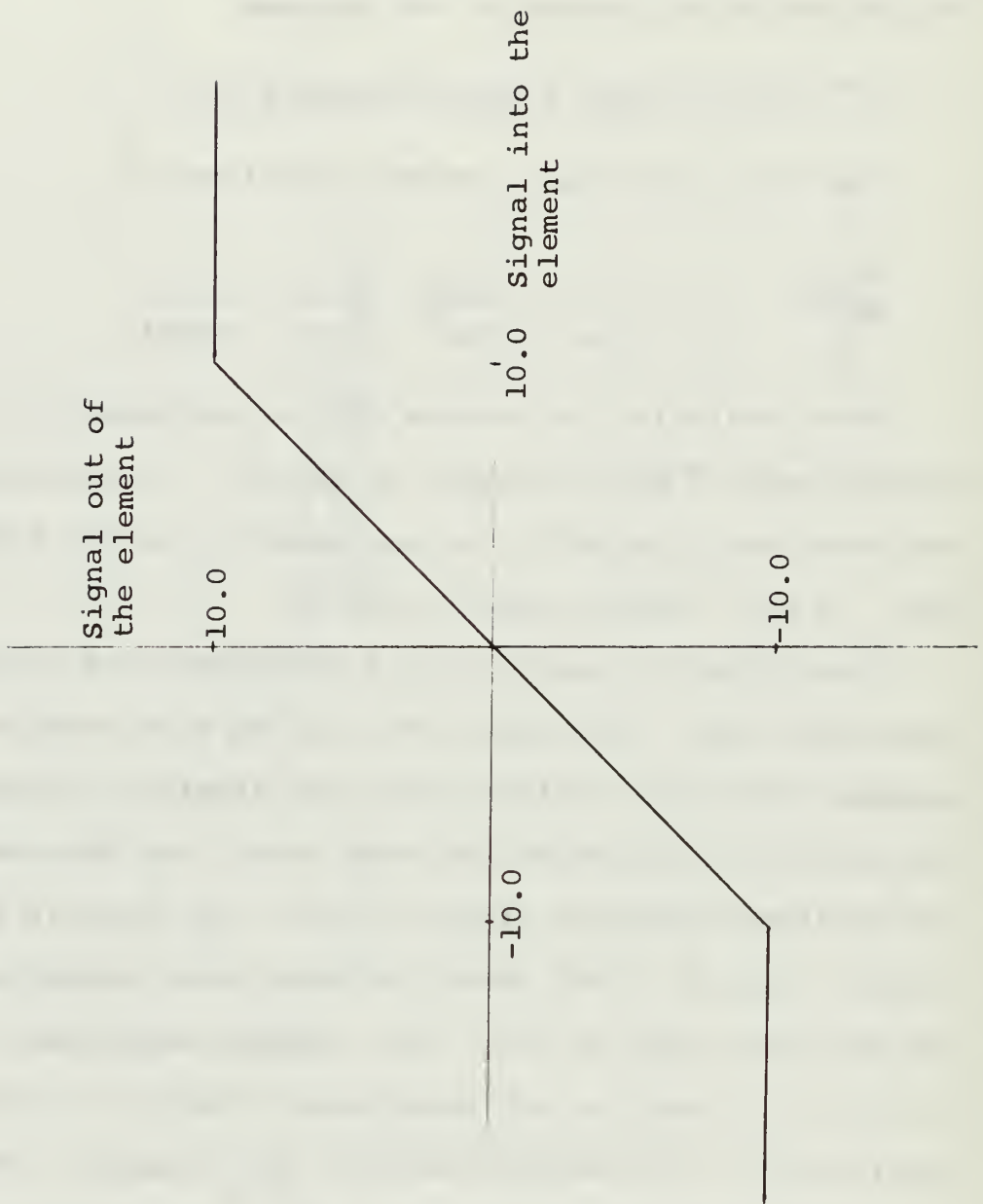
Figure 2.3a Saturation element of example in chapter 2
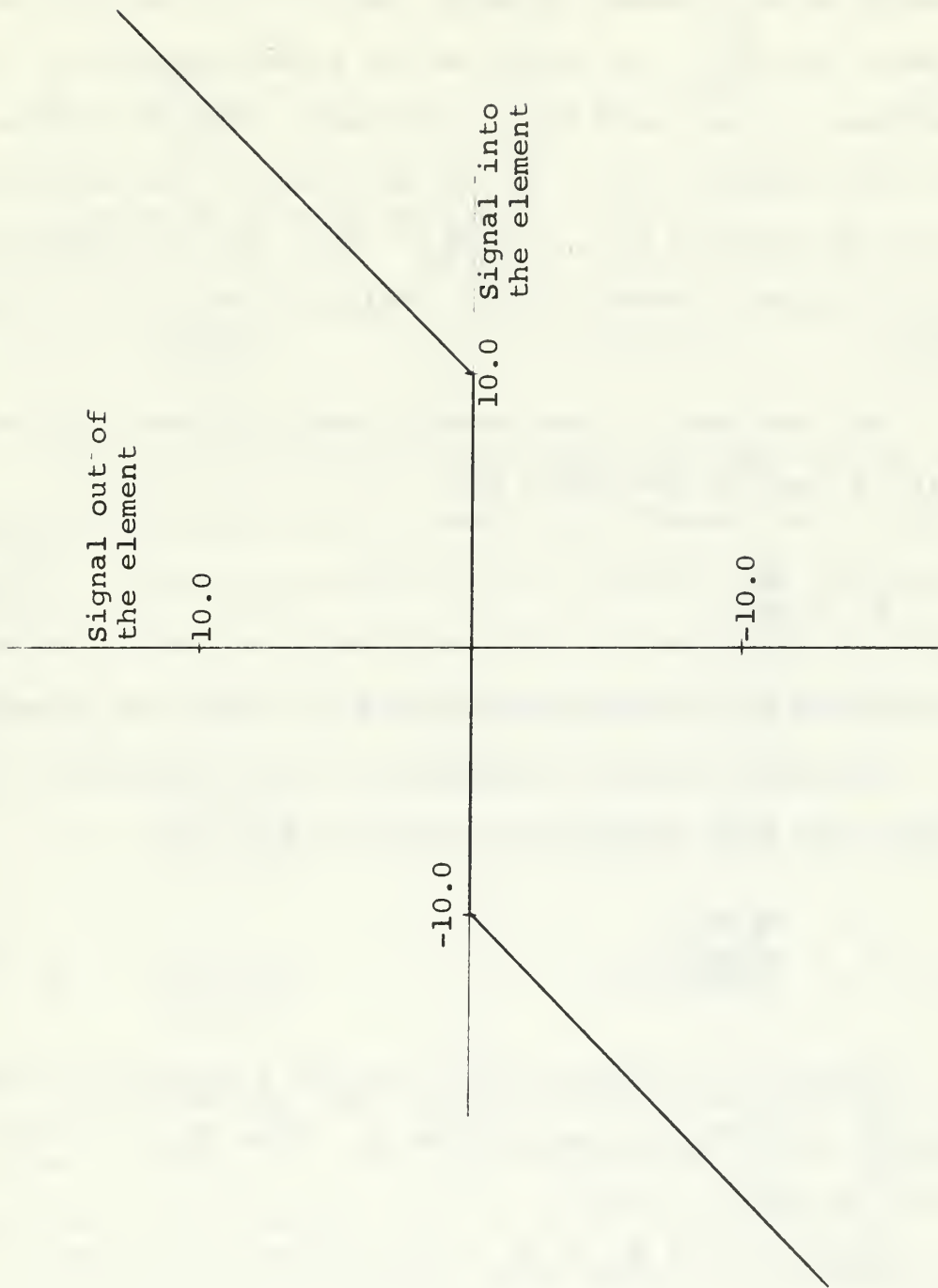
Figure 2.3b   Dead zone element in chapter 2

develop a relationship between the gain of the nonlinear element and $\frac{\Theta_{nl}}{\Theta_{in}}$, where Θ_{nl} is the signal going into the nonlinear element and Θ_{in} is the input into the system. For this example, $\Theta_{nl} = \Theta_e$ for the case of the nonlineari-ty in the forward path, and $\Theta_{nl} = \Theta_{out}$ for the nonlineari-ty in the main feedback path. This may be seen in figure (2.2).

For the case of the forward path nonlinearity, the gain, α, may be described as,

$$\alpha = \frac{\Theta_b}{\Theta_e} \tag{2.5}$$

Where Θ_b is the signal out of the nonlinear element.

Dividing both the numerator and the denominator of the right-hand side of equation (2.5) by Θ_{in},

$$\alpha = \frac{\Theta_b / \Theta_{in}}{\Theta_e / \Theta_{in}} \tag{2.6}$$

Referring to figure (2.3) for the saturation element, $\Theta_b = 10$ when Θ_e is greater than 10. When $\Theta_e \leq 10$, $\Theta_b = \Theta_e$ and α is equal to unity.

Substituting $\Theta_{in} = 20$ into equation (2.6) yields,

$$\alpha = \frac{\Theta_b / 20}{\Theta_e / \Theta_{in}} \tag{2.7}$$

24

Two cases now exist. When $\Theta_e \leq 10$, $\Theta_e = \Theta_b$ and

$$\alpha = \frac{\Theta_e/20}{\Theta_e/20} = 1.0 \qquad\qquad \Theta_e \leq 10 \qquad\qquad (2.8a)$$

When $\Theta_e > 10$, $\Theta_b = 10$ and

$$\alpha = \frac{.5}{\Theta_e/\Theta_{in}} \qquad\qquad \Theta_e \geqslant 10 \qquad\qquad (2.8b)$$

From equations (2.8a) and (2.8b), a table may be drawn up giving the gain of the saturation element as a function of $\frac{\Theta_e}{\Theta_{in}}$. This table would be the same for the saturation element in the main feedback path, by replacing Θ_e by Θ_{out}, Θ_b by Θ_a and α by β. Equations (2.8a) and (2.8b) become,

$$\beta = \frac{\Theta_{out}/20}{\Theta_{out}/\Theta_{in}} = 1.0 \qquad\qquad \Theta_{out} < 10.0 \qquad\qquad (2.9a)$$

$$\beta = \frac{.5}{\Theta_{out}/\Theta_{in}} \qquad\qquad \Theta_{out} \geq 10.0 \qquad\qquad (2.9b)$$

This table for the saturation element is table (II.la). To use it, enter the left-hand column with $\frac{\Theta_e}{\Theta_{in}}$ or $\frac{\Theta_{out}}{\Theta_{in}}$ and read the corresponding value of α or β respectively.

In like manner the dead zone device can be dealt with. Referring once again to figure (2.2)

$$\alpha = \frac{\Theta_b}{\Theta_e} \qquad\qquad (2.5)$$

Dividing again by Θ_{in} and substituting,

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} = \frac{\Theta_b/20}{\Theta_e/\Theta_{in}} \qquad\qquad (2.6),(2.7)$$

Looking at figure (2.3b), it is seen that for all input signals into the dead zone element, in this case Θ_e, less than 10.0, the output, Θ_b, is zero. Therefore,

$$\alpha = \frac{0.0/20.}{\Theta_e/\Theta_{in}} = 0.0 \qquad\qquad \Theta_e \le 10.0 \qquad (2.10a)$$

And for Θ_e greater than 10.0,

$$\alpha = \frac{\Theta_b/20}{\Theta_e/\Theta_{in}} \qquad\qquad \Theta_e \ge 10.0 \qquad (2.10b)$$

For the dead zone in the main feedback path, substitution once again obtains,

$$\beta = \frac{0.0/20.}{\Theta_{out}/\Theta_{in}} = 0.0 \qquad\qquad \Theta_{out} \le 10.0 \qquad (2.11a)$$

$$\beta = \frac{\Theta_a/20.}{\Theta_{out}/\Theta_{in}} \qquad\qquad \Theta_{out} > 10.0 \qquad (2.11b)$$

From equations (2.11a) and (2.11b) and figure (2.3b),

$\dfrac{\Theta_e}{\Theta_{in}}$ or $\dfrac{\Theta_{out}}{\Theta_{in}}$	Saturation Element α or β	Dead Zone Element α or β
0.00	1.000	0.000
0.25	1.000	0.000
0.50	1.000	0.000
0.75	0.670	0.333
1.00	0.500	0.500
1.25	0.400	0.600
1.50	0.333	0.666
1.75	0.28	0.715
2.00	0.25	0.750
2.25	0.22	0.778
2.50	0.20	0.800
2.75	0.18	0.817
3.00	0.17	0.834
3.25	0.15	0.846
3.50	0.14	0.857
3.75	0.13	0.868
4.00	0.125	0.875
4.25	0.118	0.880
4.50	0.112	0.887
4.75	0.105	0.894
5.00	0.100	0.900
5.25	0.095	0.905
5.50	0.091	0.910
5.75	0.087	0.914
6.00	0.083	0.917
6.25	0.080	0.920
6.50	0.077	0.923
675	0.074	0.926
7.00	0.071	0.929
7.50	0.067	0.932
8.00	0.063	0.938
8.50	0.059	0.941
9.00	0.055	0.945
9.50	0.053	0.948
10.00	0.050	0.950

Table II a & b a.) α and β as Functions of $\dfrac{\Theta_e}{\Theta_{in}}$ for the Saturation Element b.) α and β as Functions of $\dfrac{\Theta_{out}}{\Theta_{in}}$ for the Dead Zone Element

a table similar to table (II.1a) can be drawn up for the dead zone element. This table is table (II.1b). Once again enter with either $\frac{\Theta_e}{\Theta_{in}}$ or $\frac{\Theta_{out}}{\Theta_{in}}$ and α or β can be read off respectively.

The next step in obtaining the frequency response is to find a relationship between the nonlinear gain, α or β, and the frequency. A relationship between the nonlinear gain and the signal into the element was just shown. Using the PARAMS program, a relationship between α or β and frequency can be found. The PARAM-5 part of the program does this. Given a transfer function, it will draw a plot of gain, α or β, versus the magnitude of the transfer function with constant frequency lines drawn.

The case of the nonlinearity in the forward path will be considered first. In this case $\beta = 1.0$ and $\Theta_a = \Theta_{out}$. Looking at figure (2.2),

$$\Theta_e = \Theta_{in} - \Theta_{out} \qquad (2.12)$$

Divide both sides of equation (2.12) by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}} \qquad (2.13)$$

$\frac{\Theta_{out}}{\Theta_{in}}$ has already been computed as equation (2.4)

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha\beta} \qquad (2.4)$$

Substituting equation (2.4) into equation (2.13) and letting $\beta = 1.0$.

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^4 + 18s^3 + 95s^2 + 150s}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \qquad (2.14)$$

Using equation (2.14) and letting α vary between 0.0 and 1.5, a plot as described in the previous paragraph is obtained by the PARAM-5 subprogram of the PARAMS program.

This plot is figure (2.4), with α, the nonlinear gain in the forward path as the ordinate and $\frac{\Theta_e}{\Theta_{in}}$ as the abscissa. Constant ω lines, $\omega = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6,$ 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2 and 3.4, were drawn by the PARAMS program. Using table (II.1a) and table (II.1b), constant Θ_{in} lines are drawn on figure (2.4) for the saturation element and for the dead zone element. The constant Θ_{in} lines are drawn using α and $\frac{\Theta_e}{\Theta_{in}}$ as the coordinates.

After drawing the constant Θ_{in} curves on figure (2.4), it is seen that these curves intersect the constant omega curves in several places. The intersection of the constant Θ_{in} curves with the constant omega curves, with the associated value of α at each intersection, determines a relationship between α, the nonlinear gain, and ω, the frequency of the overall system. With this relationship between α and ω, the total frequency response will be obtained. Figure (2.4) shows a pair of relationships, one

29
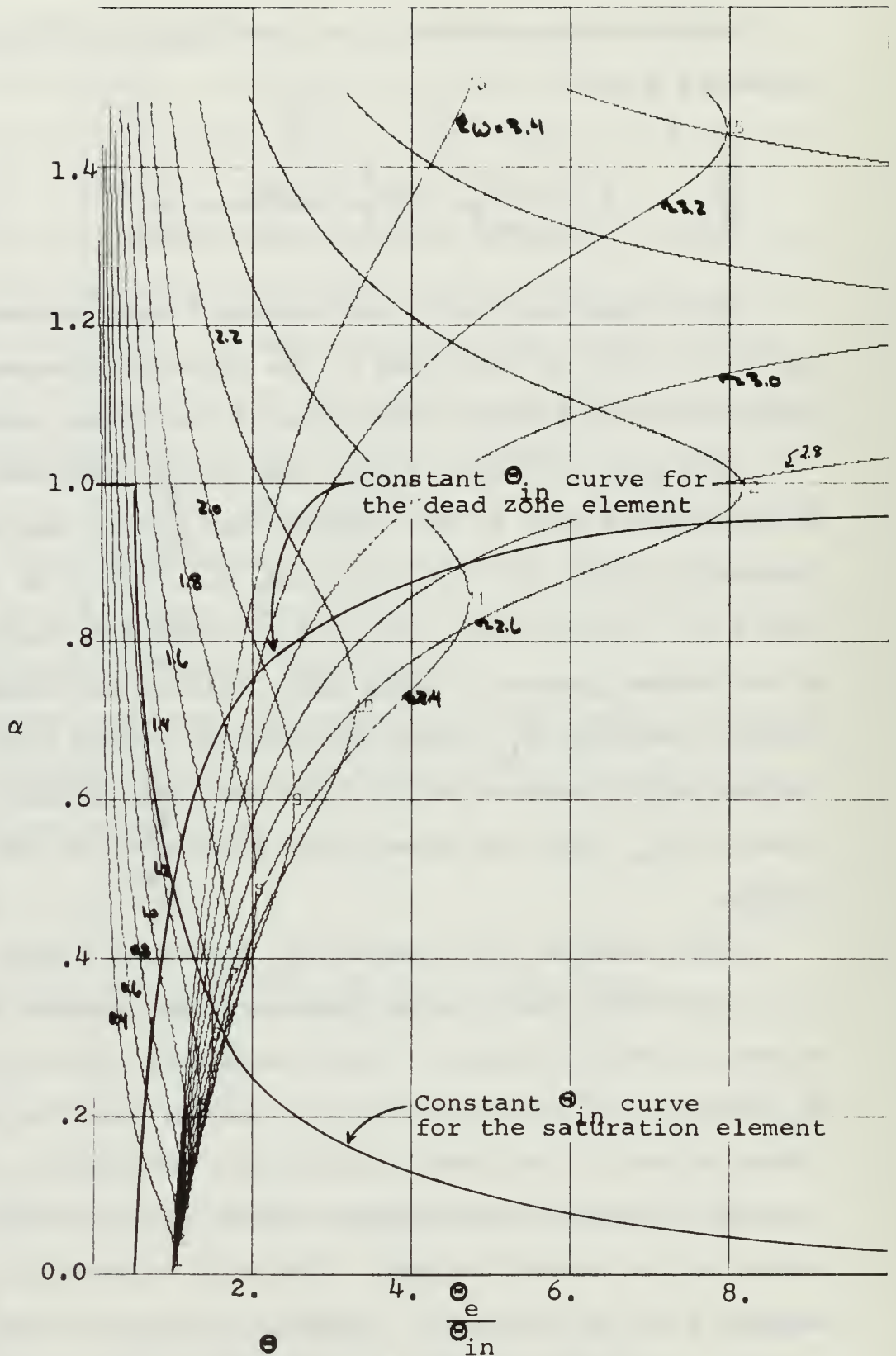
Figure 2.4 α versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for a nonlinearity in the forward path

for the saturation element and the other for the dead zone element. These two are in no way related to each other. They are both shown on the same plot so as to analyze two different cases at one time.

The final step in the analysis is obtaining the frequency response itself. Once again the PARAMS program is used. The PARAM-7 subprogram of the PARAMS program gives a frequency response plot with constant α or β lines drawn on it. Because (for this analysis) the closed loop frequency response is desired, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude (dB) versus frequency with constant α curves is obtained. This is figure (2.5) with α curves of $\alpha = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4,$ and 1.5 drawn on it.

Now using the relation that was found in figure (2.4) between α and ω through the intersection of the constant ω curves and the constant ω curves, these points are plotted on figure (2.5). The coordinates to transfer these points from figure (2.4) to figure (2.5) are α and ω. Where there is no exact α or ω line on figure (2.5), interpolation is used.

When looking at figure (2.5) it should be noted that the saturation frequency response shows some jump resonance around its resonance peak. This is characteristic of saturation elements and can be predicted from figure (2.4).
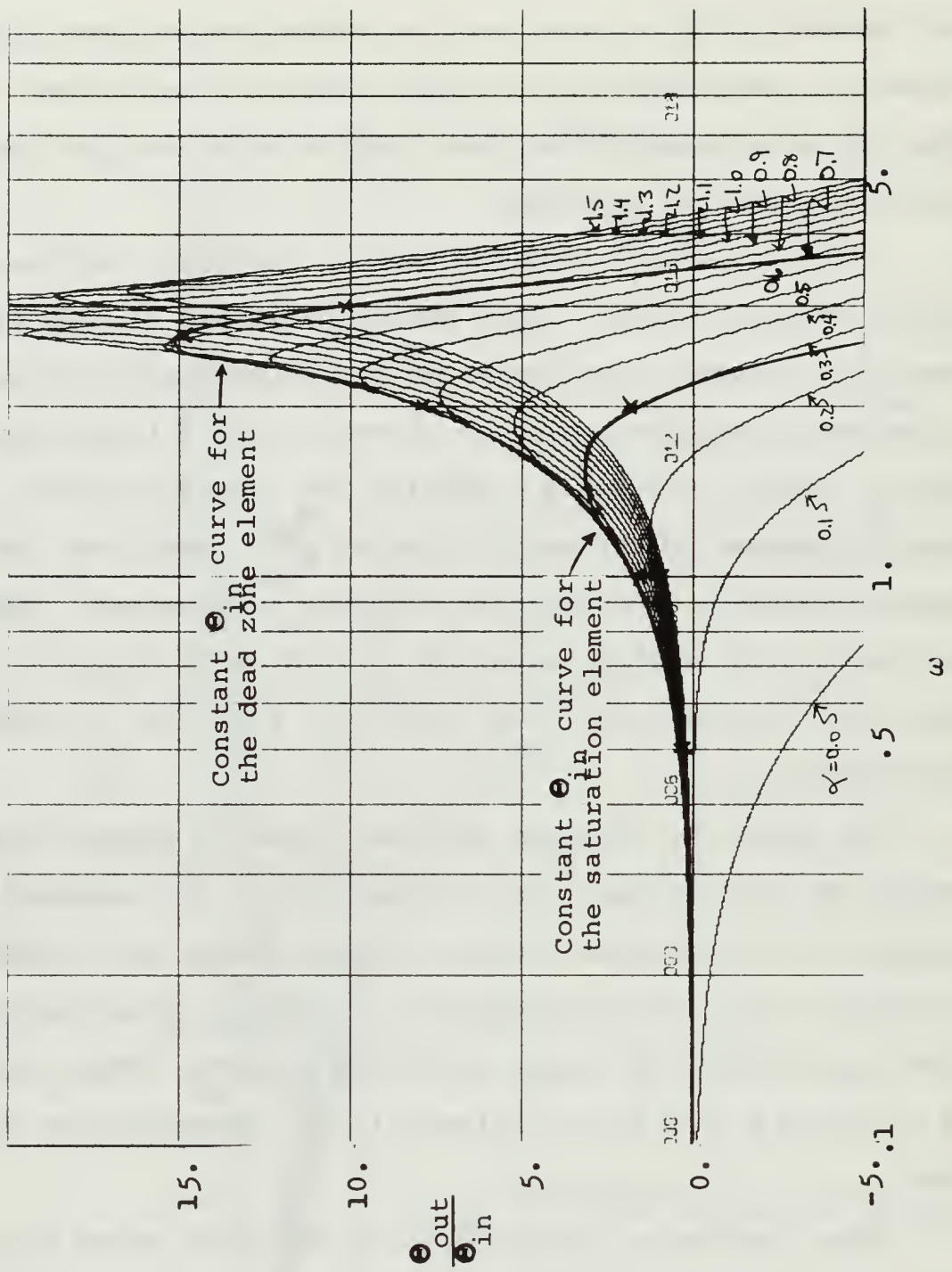
Figure 2.5 Closed loop frequency response of the servo-
mechanism with a nonlinearity in the forward path

32

To do this, trace the constant Θ_{in} saturation curve in figure (2.4) from left to right. Note that it intersects the constant $\omega = 1.4$ curve twice giving two values of non-linear gain, α, for $\omega = 1.4$. This is part of the jump resonance effect and will be looked at more closely in Chapter 4.

To test the results of this graphical analysis, an analog simulation of the system was performed and the actual frequency response was obtained. How this simulation was performed and how the nonlinear elements were simulated is explained in Appendix A. The analog results are plotted on figure (2.5) as 'x'. The analog simulation was close enough to justify the conclusion that the graphical analysis was accurate in predicting the frequency response. Any difference between the predicted response and actual response is due to errors in graphical construction, reading of the graph and tolerance in the simulating equipment.

The main effort of this thesis is directed to closed loop frequency response. However, the open loop frequency response may be obtained in the same manner as the closed loop response. The means of obtaining the open loop response is in the final step of the analysis. When using the PARAM-7 subroutine of the PARAMS program, instead of using the closed loop transfer function, the open loop transfer function should be used. The plot that should be

obtained is the open loop magnitude (dB) versus frequency

with constant α curves drawn on the plot by the program.

For this first case the plot is figure (2.6). The phase

curve is superimposed for more information. The next step

is the same as in the first case. Using the coordinate

values of α and ω found in figure (2.4), redraw the con-

stant Θ_{in} curves on figure (2.6). The result is the open

loop frequency response for a system with either saturation

or dead zone in the forward path.

The next case in this example is a nonlinear element

in the main feedback path. In this case $\alpha = 1.0$ and Θ_{out}

is the signal into the nonlinear element as shown in figure

(2.2). Equations (2.9a), (2.9b), (2.11a), and (2.11b) have

been developed for the nonlinear element in the main feed-

back path and the results tabulated in table (2.1a) for a

saturation element and table (2.1b) for a dead zone element.

Using the PARAM-5 subprogram of the PARAMS program, a

plot of β, the nonlinear gain in the main feedback path,

versus the magnitude of $\dfrac{\Theta_{out}}{\Theta_{in}}$ is obtained in the same manner

as for figure (2.4). This plot is figure (2.7) and shows β

between 0.0 and 1.5 as the ordinate and $\dfrac{\Theta_{out}}{\Theta_{in}}$ as the abscissa

with constant ω curves of $\omega = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4,$

1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 3.0, 3.2, and 3.4 drawn by

the program on the plot.
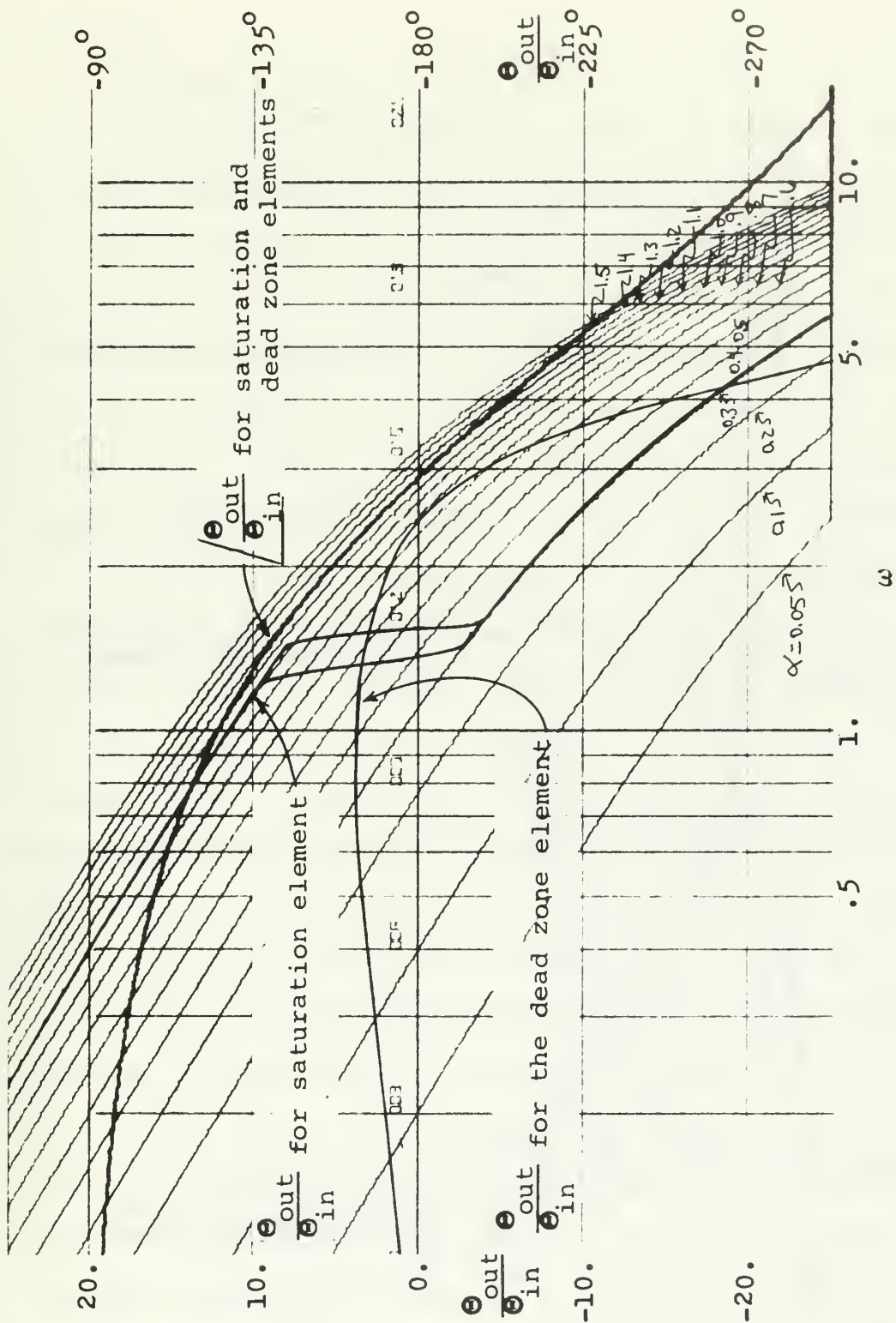
Figure 2.6   Open loop frequency response for a saturation
and dead zone elements in the forward path

35
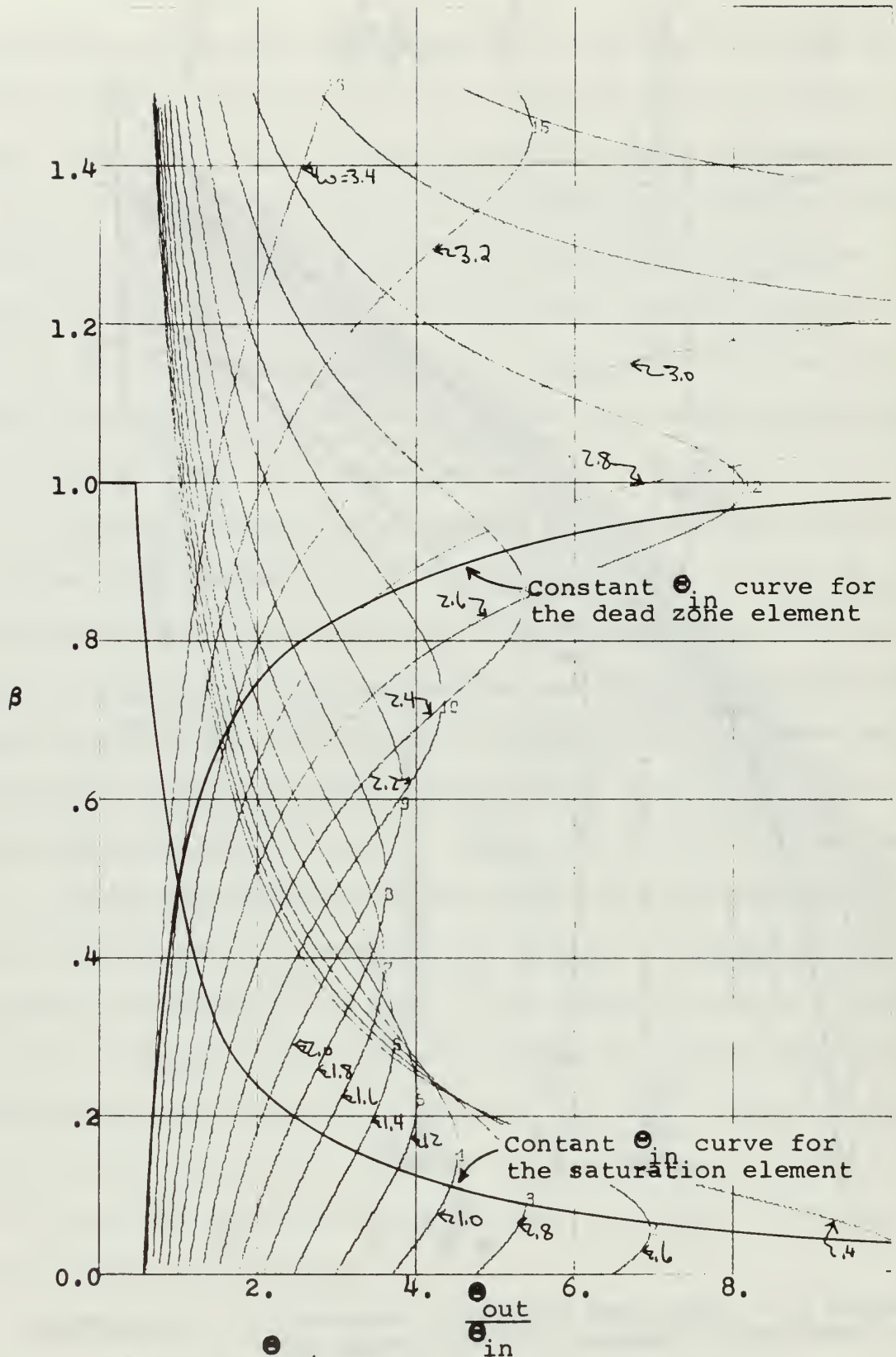
Figure 2.7 β versus $\dfrac{\theta_{out}}{\theta_{in}}$ magnitude for a nonlinearity in the feedback path

36

The next step in the analysis is the same as in the first case. Using the values in table (II-la) and table (II-lb), constant Θ_{in} curves for saturation and dead zone are drawn on figure (2.7). The coordinates for drawing these two curves on figure (2.6) are β and $\frac{\Theta_{out}}{\Theta_{in}}$. The intersection in figure (2.6) of the constant Θ_{in} curves with the constant ω curves with an associated β for each intersection, gives the relationship between the nonlinear gain, β and ω for the system.

The final step in this analysis of the second case is the obtaining of the closed loop frequency response. As in the first case, the PARAM-7 subprogram of the PARAMS program is used. Using equation (2.4) with $\alpha = 1.0$ figure (2.8) is obtained with the magnitude of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) as the ordinate and frequency as the abscissa. Constant β lines of $\beta = 0.0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 are drawn on the plot by the program.

Using the coordinate of the intersection on the constant Θ_{in} curves and the constant ω curves, that is β and ω, the constant Θ_{in} curves are drawn on figure (2.8). These curves on figure (2.8) give the closed loop frequency response of the overall system for a saturation element in the main feedback path and for a dead zone element. Note that there is no jump resonance in either response. This is as predicted by looking at figure (2.7) and observing that neither constant Θ_{in} curves intersects a constant omega

37
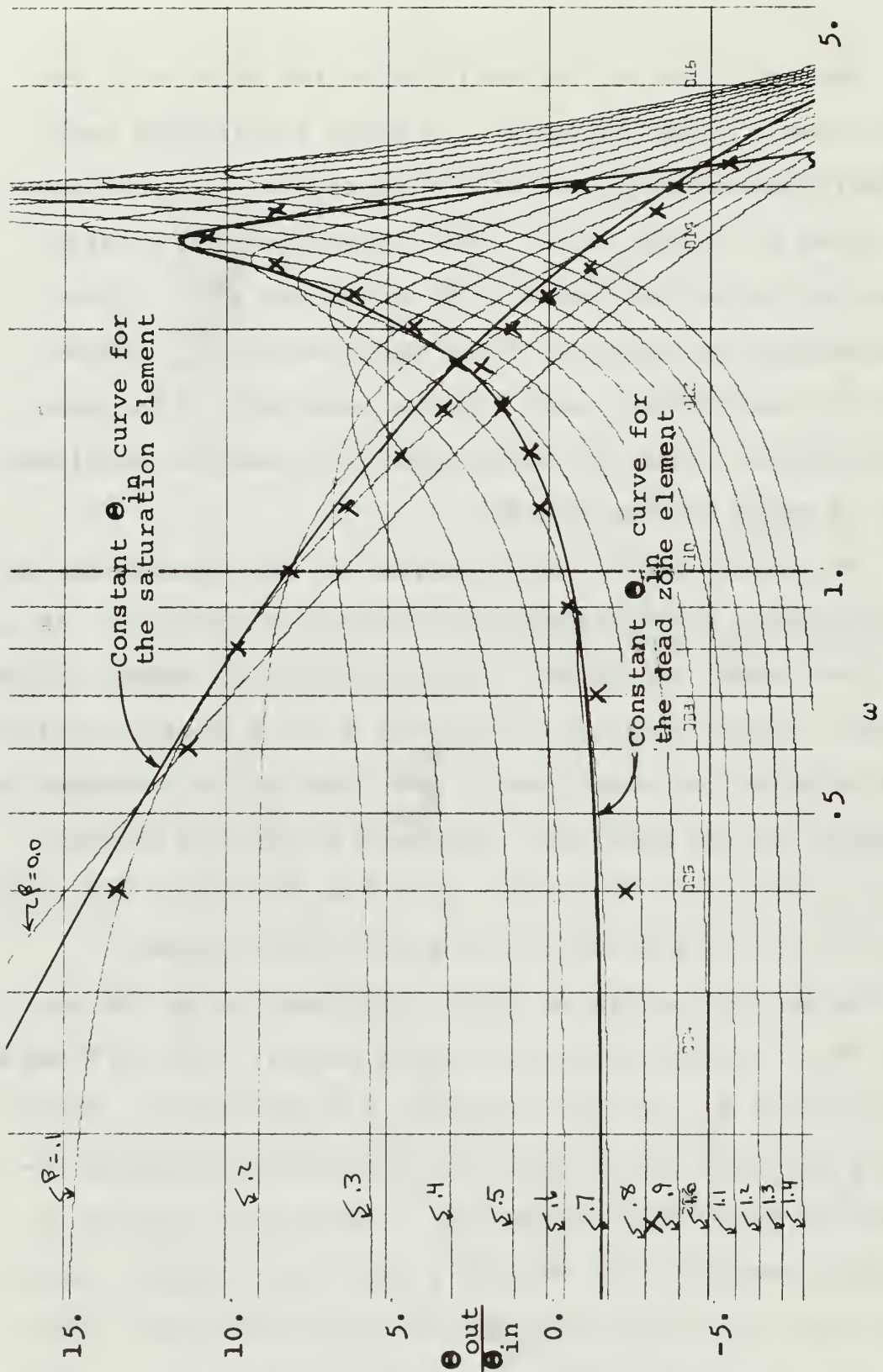
Figure 2.8 Closed loop frequency response for servo-
mechanism with a nonlinearity in the feedback path

38

curve more than once. This could have occurred in both cases, saturation and dead zone, if the constant Θ_{in} curves had been shifted sufficiently to the right in figure (2.7). The shifting to the right of the constant Θ_{in} saturation curve corresponds to raising the value at which the signal saturates. The shifting of the constant Θ_{in} dead zone curve to the right corresponds to increasing the area of dead zone.

The results in this second case, a nonlinearity in the main feedback path, were also checked by analog simulation. The simulation results are plotted with 'x' and correspond enough to verify that the graphical analysis is correct.

C.   STEPS IN THE ANALYSIS

The steps in the analysis are listed below for ease in following the process. The use of the PARAMS program makes this analysis realizable because of its speed and ease of calculation. At the present time, the PARAMS program is limited to a 49th order system.

1.   Draw the block diagram of the system, with the nonlinear element as a separate block. In order to use this technique, the nonlinearity must be a single-valued type.

2.   Write the transfer function of the closed loop system. Also write the transfer function of the signal preceding the nonlinearity with respect to the input signal

39

into the system. In some cases these two transfer func-
tions may be the same as in the case in this chapter where
the nonlinearity was in the main feedback path.

3. Using the PARAMS program, or its equivalent, ob-
tain a plot of the nonlinear gain versus the magnitude
transfer function of the signal into the nonlinearity.
This plot should have drawn on it constant ω curves for
the frequencies of interest.

4. Obtain a relationship between the nonlinear gain
and magnitude of the transfer function of the signal into
the nonlinearity. This can be done several ways, one of
which is described in this chapter.

5. Plot this relationship found in step 4 onto the
plot drawn in step 3 using the nonlinear gain and transfer
function magnitude as the coordinates. This is the constant
Θ_{in} curve.

6. The intersection of the constant Θ_{in} curve and the
constant ω curves gives an associated value of nonlinear
gain for each intersection. These intersections give the
relationship between the nonlinear gain and frequency. In
some cases there may be more than one associated value of
gain for a certain frequency. This can be ascribed to a
jump resonance effect.

7. Again using the PARAMS program or its equivalent,
obtain a standard frequency response plot, either open or
closed loop, with constant nonlinear gain curves drawn on

40

it. The values of nonlinear gain curves must correspond to the range of nonlinear gain found in step 6.

8. Using the values of nonlinear gain and frequency found in step 6 as coordinates, replot the constant Θ_{in} curve on the plot found in step 7. This gives the closed or open loop frequency response - whatever the case may be - of a system with a nonlinear element to a constant magnitude input signal.

D.   FURTHER INVESTIGATION OF THE JUMP RESONANCE EFFECT

The frequency response for the system with saturation in the forward path, figure (2.5), shows a jump resonance around $\omega = 1.4$. This jump resonance is to be expected with a nonlinear device. In Chapter 4 this effect will be shown for a dead zone element. To illustrate this jump resonance more clearly, another example will be shown here. The example is a second order feedback system which is used as an example in a nonlinear control textbook.[2] Its block diagram is figure (2.9).

By letting α represent the variable gain of the saturation element, the closed loop transfer function may be written as,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{200\alpha}{s^2 + s + 200\alpha} \tag{2.15}$$

---

[2] Thaler, G. J. and Pastel, M.P., <u>Analysis and Design of Nonlinear Feedback Control Systems</u>, p. 198, McGraw-Hill, 1962.
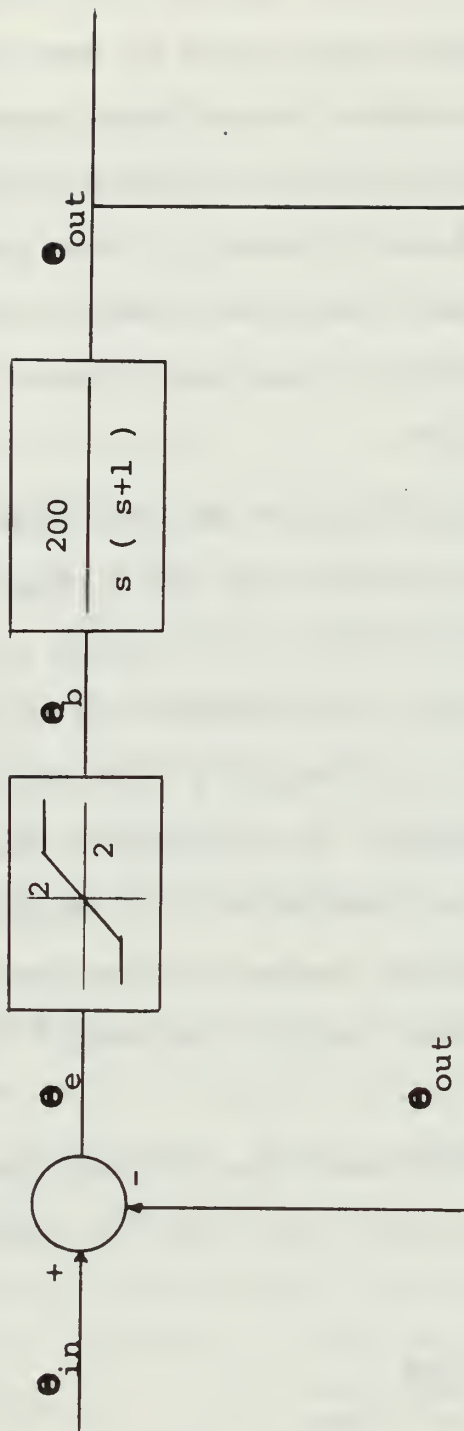
Figure 2.9 Block diagram of a servo with saturation

What is desired is the frequency response with a constant input magnitude. Θ_{in} is chosen to be $\Theta_{in} = 2.0$.

Following the steps in the analysis technique, the transfer function of the signal preceding the nonlinear element with respect to Θ_{in}, that is $\dfrac{\Theta_e}{\Theta_{in}}$, is derived

$$\Theta_e = \Theta_{in} - \Theta_{out} \qquad\qquad (2.16)$$

Dividing by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}} \qquad\qquad (2.17)$$

And substituting equation (2.15) into equation (2.17),

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^2 + s}{s^2 + s + 200\alpha} \qquad\qquad (2.18)$$

By using the PARAMS-5 subprogram of the PARAMS and equation (2.18), a plot of α versus $\dfrac{\Theta_e}{\Theta_{in}}$ with constant ω curves is obtained. This plot is figure (2.10).

Next the relation between $\dfrac{\Theta_e}{\Theta_{in}}$ and α is found.

Referring to figure (2.9)

$$\alpha = \frac{\Theta_b}{\Theta_e}$$

Dividing both the numerator and denominator of the right-hand side of equation (2.19) by Θ_{in},

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} \qquad\qquad (2.20)$$

43
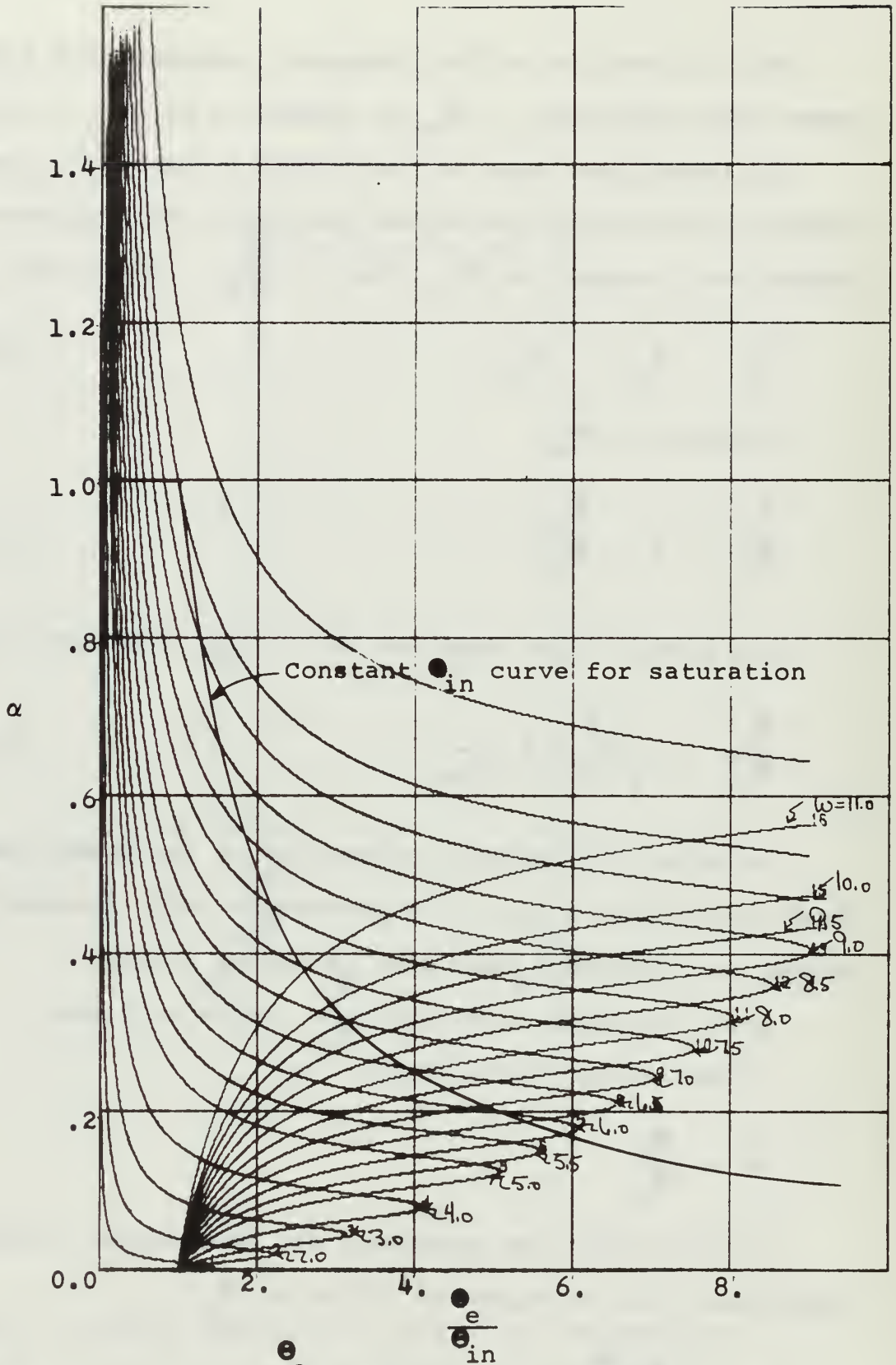
Figure 2.10 α versus $\dfrac{\theta_e}{\theta_{in}}$ magnitude for servo with saturation

44

and when $\Theta_e \leqslant 2.0$, $\alpha = 1$ and when $\Theta_e \geqslant 2.0$, $\frac{\Theta_e}{\Theta_b} = 2.0$ as the element goes into saturation. Two cases exist for $\Theta_{in}=2.0$.

$$\alpha = 1.0 \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \leq 1.0 \qquad\qquad (2.21a)$$

$$\alpha = \frac{1.0}{\Theta_e / \Theta_{in}} \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \geq 1.0 \qquad\qquad (2.21b)$$

With equations (2.21a) and (2.21b), a table is constructed giving α for the corresponding values of $\frac{\Theta_e}{\Theta_{in}}$. This table is table (II.2) where $0.0 \leq \frac{\Theta_e}{\Theta_{in}} \leq 10.0$.

Using the values in the table, the constant Θ_{in} curve is plotted on figure (2.10). The intersection of this constant Θ_{in} curve and the ω curves in figure (2.10) gives the relation between ω and α for a constant $\Theta_{in} = 2.0$.

The closed loop frequency response of the servo can now be obtained for a $\Theta_{in} = 2.0$. Using the PARAM-7 subprogram and equation (2.15), a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω is obtained. This is figure (2.11) and has constant α curves of $\alpha = 0.05$, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7, 0.8, 0.9 and 1.0 plotted on it. Using the coordinates of α and ω for the constant Θ_{in} curve in figure (2.10), the constant Θ_{in} curve is replotted onto figure (2.11). As may be seen, there is a large jump resonance effect between $\omega = 6.0$ and $\omega = 10.0$.

The jump resonance effect can also be seen in the phase curve of this example. The technique is similar to that of drawing the magnitude frequency response. Using

45

$\dfrac{\Theta_e}{\Theta_{in}}$	α
0.0	1.0
0.5	1.0
1.0	1.0
1.5	0.667
2.0	0.500
2.5	0.400
3.0	0.333
3.5	0.286
4.0	0.250
4.5	0.222
5.0	0.200
5.5	0.182
6.0	0.167
6.5	0.154
7.0	0.143
7.5	0.133
8.0	0.125
8.5	0.118
9.0	0.111
9.5	0.105
10.0	0.100

Table II.2 α as a Function of $\dfrac{\Theta_e}{\Theta_{in}}$ for the Saturation Element
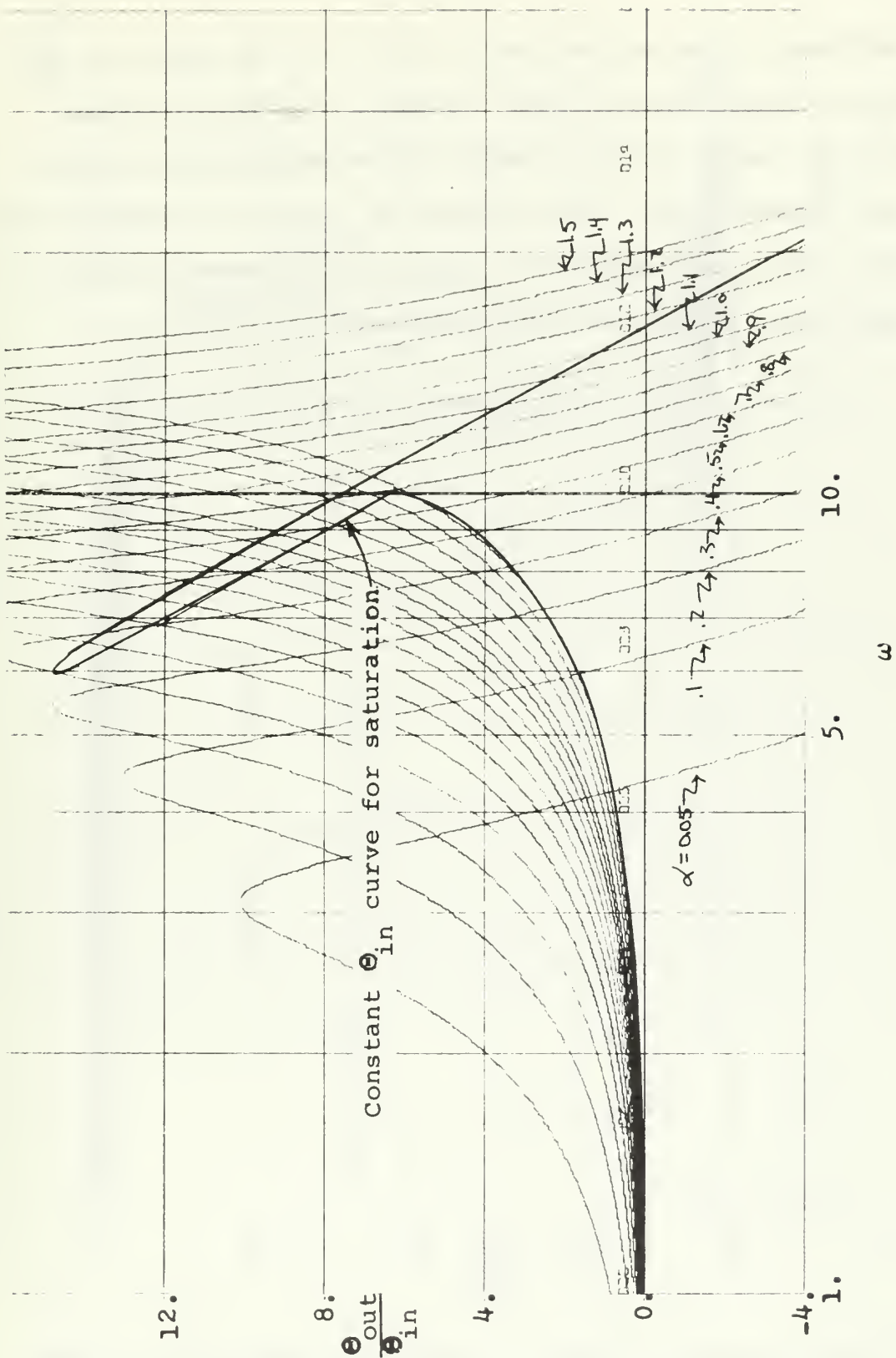
Figure 2.11 Closed loop magnitude frequency response for a
a servo with saturation

47

the PARAM-7 subprogram again, a plot of the phase of $\frac{\Theta_{out}}{\Theta_{in}}$ versus omega with the same constant α curves is drawn. This is figure (2.12). Using the coordinates of α and ω from figure (2.10), the constant Θ_{in} curve is drawn on the phase plot, figure (2.12). The jump resonance effect is seen again, though not as pronounced.
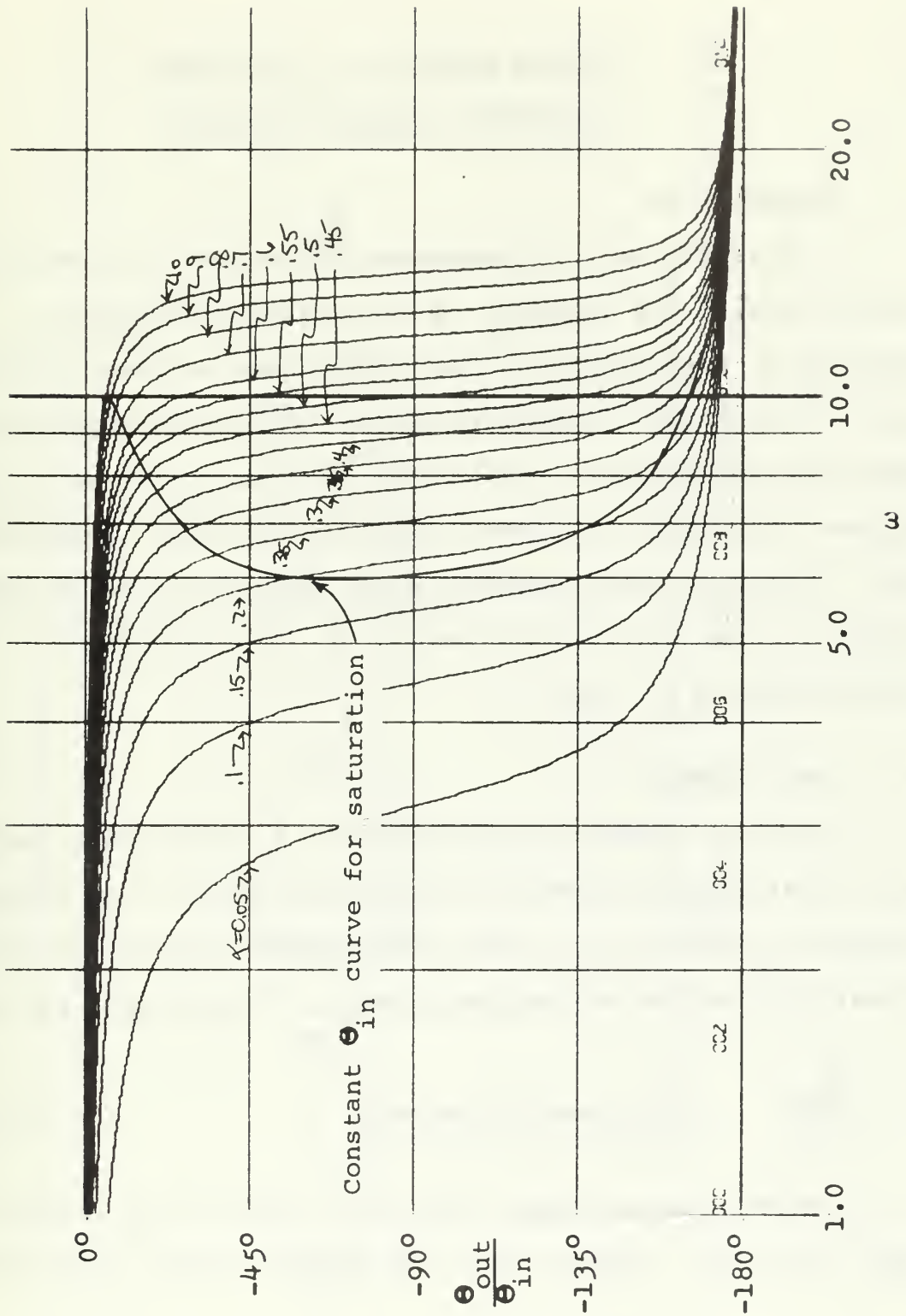
Figure 2.12 Closed loop phase frequency response for the
servo with saturation

49

III. FURTHER EXAMPLES OF NONLINEAR
FREQUENCY RESPONSE ANALYSIS

A. INTRODUCTION

In this chapter, six examples are worked in order to further verify the validity of the analysis technique. Saturation, dead zone, an ideal relay, and coulomb frction plus stiction are the nonlinearities used in the examples. These nonlinearities are placed in various positions in the systems including the forward path and the minor feedback path. The first four examples were checked by analog simulation. Also in all six examples Θ_{in} was assumed to be of magnitude $\Theta_{in} = 10.0$.

B. FIRST EXAMPLE

The first example to be worked is a third order system with a saturation element in the forward path. Its block diagram is figure (3.1) where α represents the variable nonlinear gain of the saturation element. From figure (3.1),

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{9\alpha}{s^3 + s^2 + 16.25s + 9\alpha} \tag{3.1}$$

The saturation element has unity gain for Θ_e signals less than 50.0. Signals into the element greater than 50.0

θ_{out}

$\dfrac{9.0}{s(s^2+s+16.25)}$

θ_b

α

θ_e

$-$

$+$

θ_{in}
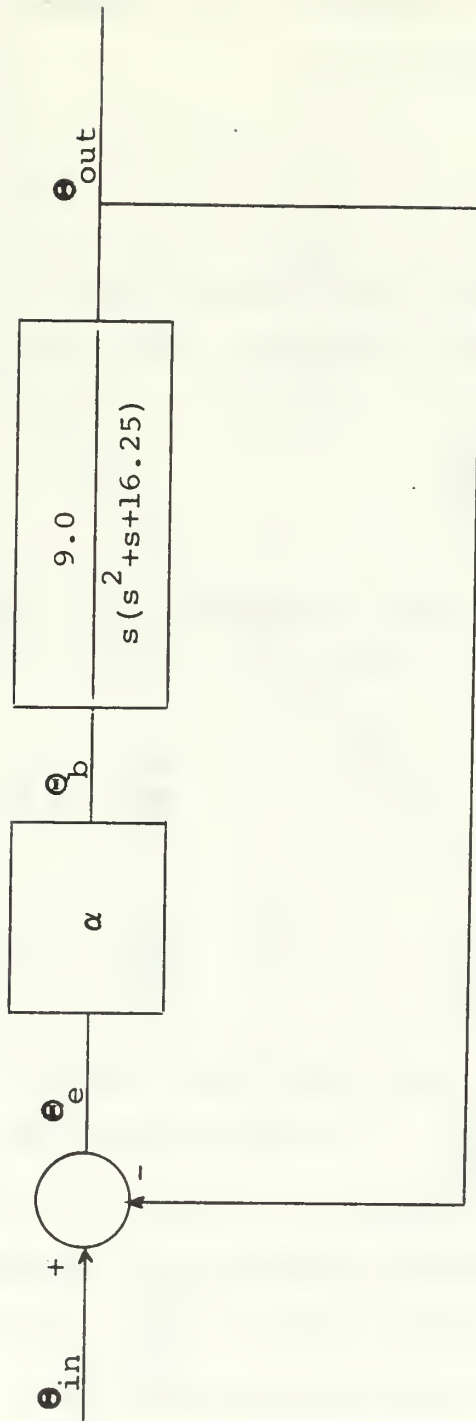
Figure 3.1 Block diagram of example #1 where ⊗ represents the variable gain of the saturation element

are limited so that the output of the element is 50.0.
The input-output diagram of the element is figure (3.2).
Referring to figure (3.1),

$$\alpha = \frac{\Theta_b}{\Theta_e} \tag{3.2}$$

Dividing both the numerator and the denominator of the right-hand side of equation (3.2) by Θ_{in}

$$\alpha = \frac{\Theta_b/\Theta_{in}}{\Theta_e/\Theta_{in}} \tag{3.3}$$

For a $\Theta_{in} = 10.0$, equation (3.3) can be divided into two parts.

$$\alpha = 1.0 \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \leq 5.0 \tag{3.4a}$$

$$\alpha = \frac{5.0}{\Theta_e/\Theta_{in}} \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \geq 5.0 \tag{3.4b}$$

Equations (3.4a) and (3.4b) define the relationship between α and $\frac{\Theta_e}{\Theta_{in}}$. It was seen in the previous chapter that this relationship is necessary to obtain the frequency response. These equations are tabulated in table (II.1), where the left-hand column is $\frac{\Theta_e}{\Theta_{in}}$ and α is the value of nonlinear gain for the particular $\frac{\Theta_e}{\Theta_{in}}$.

52
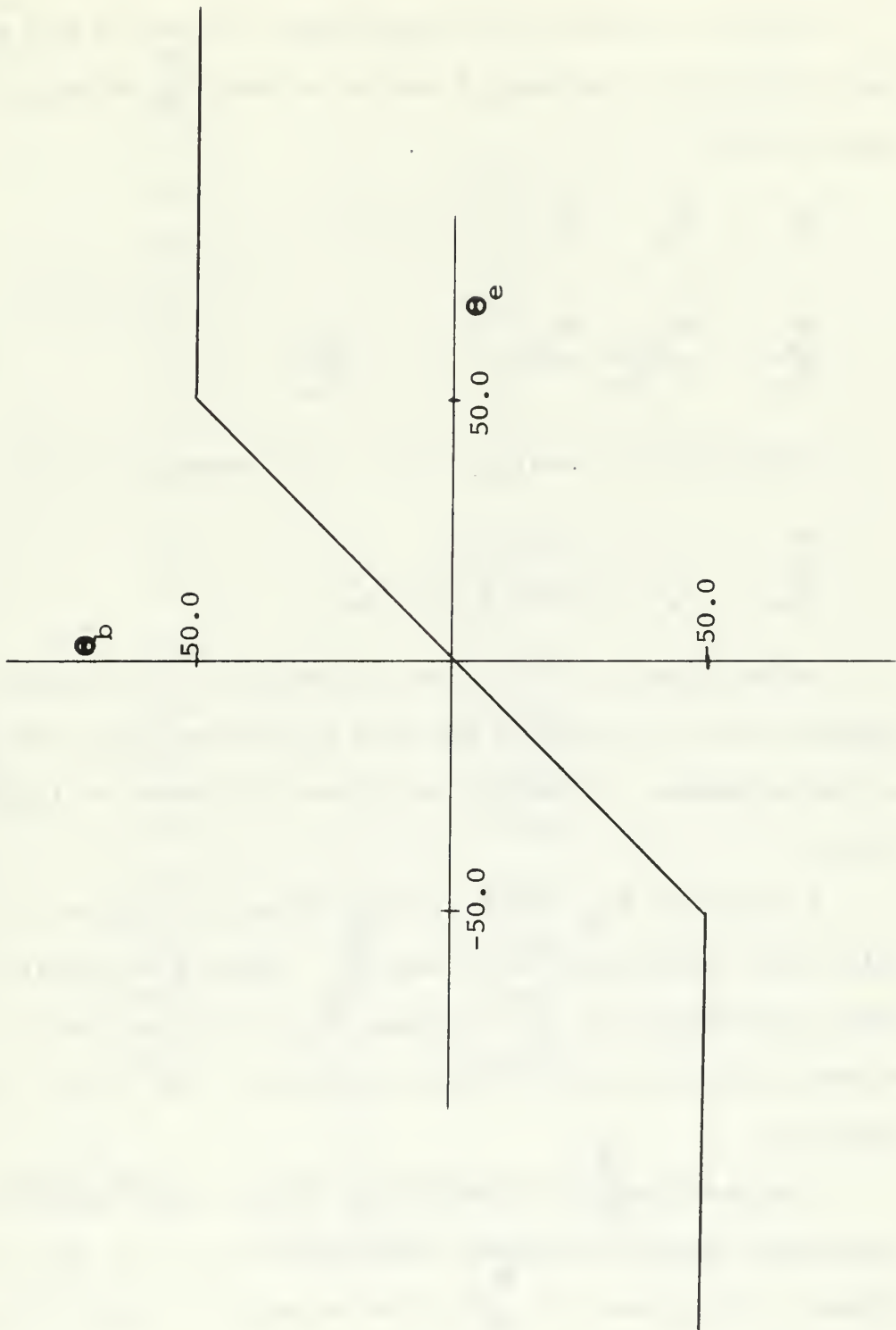
Figure 3.2 Saturation element of example #1

Having obtained the relationship between α and $\dfrac{\Theta_e}{\Theta_{in}}$, the relationship between α and ω is found. Referring to figure (3.1),

$$\Theta_e = \Theta_{in} - \Theta_{out}$$

$$\frac{\Theta_e}{\Theta_{in}} = \frac{\Theta_{in} - \Theta_{out}}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}} \tag{3.5}$$

Substituting equation (3.1) into equation (3.5),

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^3 + s^2 + 16.25}{s^3 + s^2 + 16.25 + 9\alpha} \tag{3.6}$$

Using equation (3.6) and the PARAM-5 subprogram, figure (3.3) is obtained where α is the abscissa and $\dfrac{\Theta_e}{\Theta_{in}}$ is the ordinate. Constant ω curves are drawn on figure (3.3).

A constant Θ_{in} curve is then drawn on figure (3.3) using the coordinates of α and $\dfrac{\Theta_e}{\Theta_{in}}$ taken from table (III.1). The intersection of the constant Θ_{in} curve and the ω curves gives the relationship between α and ω for this analysis.

The next step is drawing the closed loop frequency response. A plot is made, using equation (3.1) and the PARAM-7 subprogram, of $\dfrac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω. This plot is figure (3.4) and has constant α curves on it over the range of α that is used. Using the coordinates of α and ω

54

$\dfrac{\Theta_e}{\Theta_{in}}$	α
0.0	1.0
1.0	1.0
2.0	1.0
3.0	1.0
4.0	1.0
5.0	1.0
6.0	0.833
7.0	0.715
8.0	0.625
9.0	0.555
10.0	0.500
11.0	0.454
12.0	0.416
13.0	0.385
14.0	0.357
15.0	0.333

Table III.1 α as a Function of $\dfrac{\Theta_e}{\Theta_{in}}$ for the Saturation Element in Example #1
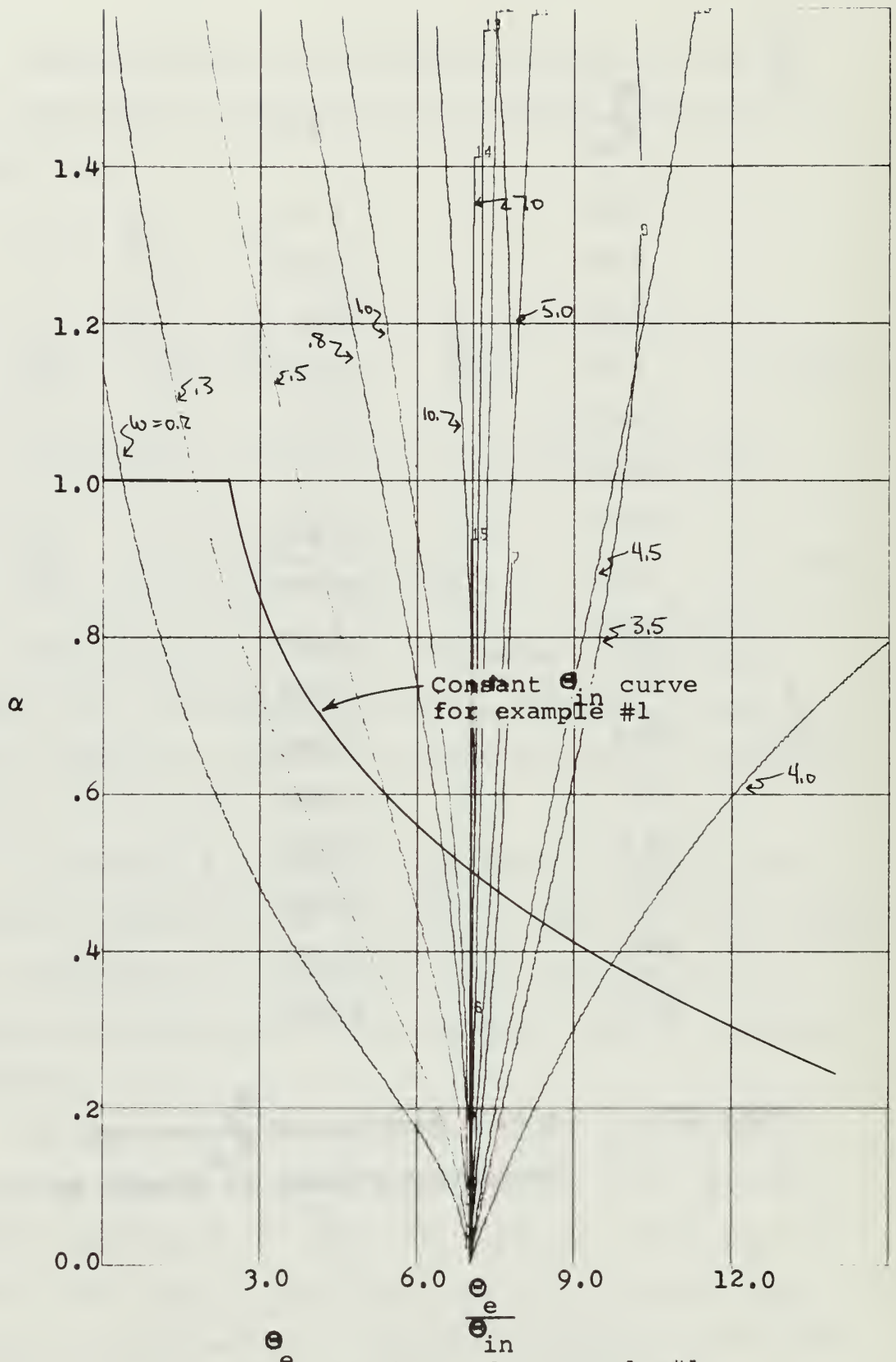
Figure 3.3 α versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for example #1

56
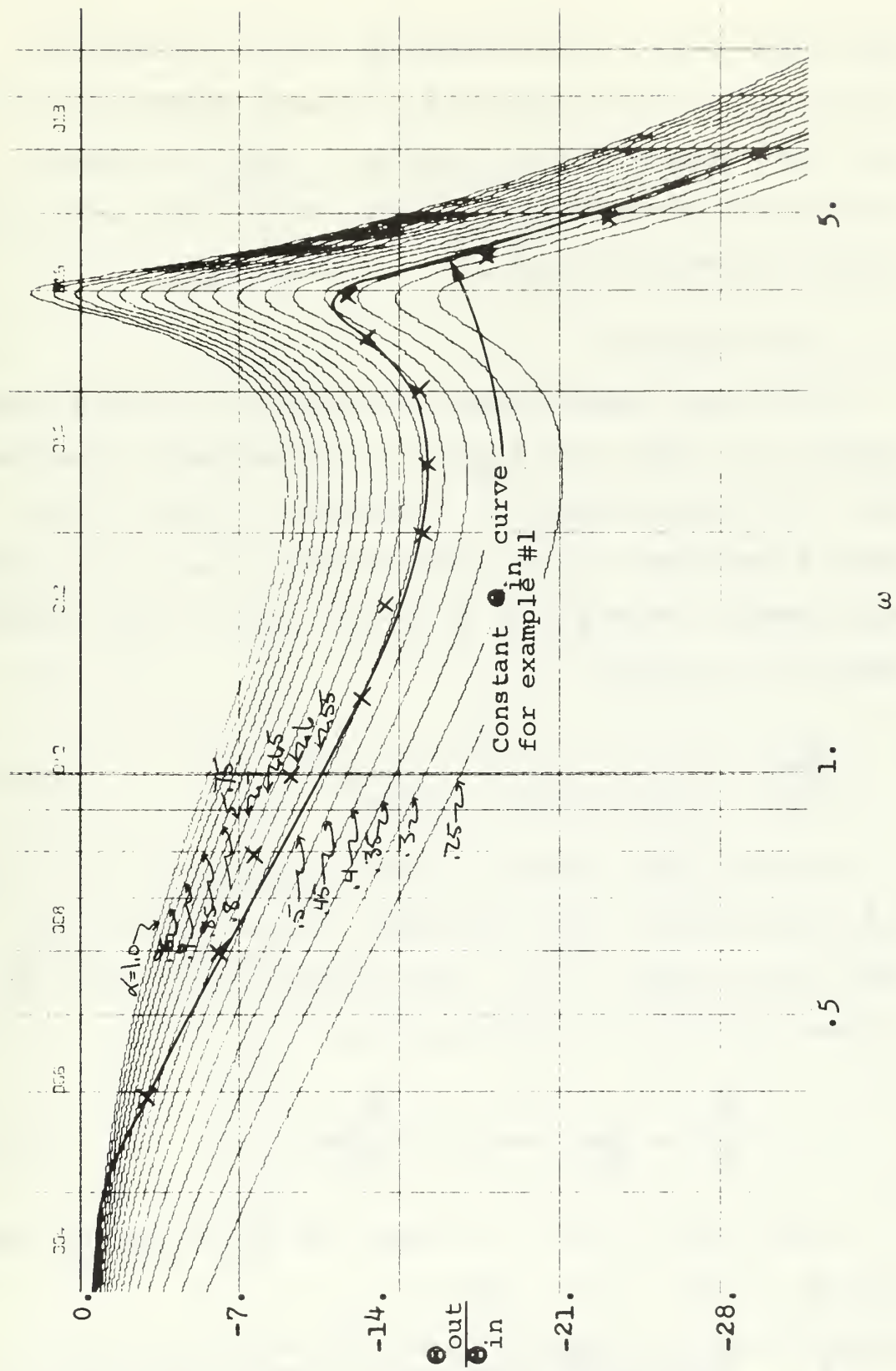
Figure 3.4 Closed loop frequency response of example #1

57

from figure (3.3), the constant Θ_{in} curve is redrawn on figure (3.4). This constant Θ_{in} curve on figure (3.4) is the closed loop frequency response of the first example with the saturation element in the system. The analog simulation is denoted by 'x'.

C. SECOND EXAMPLE

The second example used for analysis is a third order system with a dead zone element in the tachometer feedback path. The block diagram of the system is figure (3.5) where β represents the variable nonlinear gain of the dead zone element. From figure (3.5), the closed loop transfer function is written,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{9}{s^3 + 16.25s + 9 + \beta s^2} \qquad (3.7)$$

The dead zone element is shown in figure (3.6). For a Θ_a less than 4.0, Θ_d, the output, is zero. For a constant input of $\Theta_{in} = 10.0$, this condition exists when $\frac{\Theta_a}{\Theta_{in}}$ is less than 0.4. In equation form,

$$\beta = \frac{\Theta_d}{\Theta_a} = \frac{0.0}{\Theta_a} = 0.0 \qquad \frac{\Theta_a}{\Theta_{in}} \le 0.4 \qquad (3.7)$$

From figure (3.6), it is seen that for Θ_a greater than 4.0, $\Theta_d = 3 (\Theta_a - 4.0)$. Thus,

$$\beta = \frac{\Theta_d}{\Theta_a} = \frac{3(\Theta_a - 4.0)}{\Theta_a} \qquad \Theta_a \ge 4.0 \qquad (3.8)$$

58
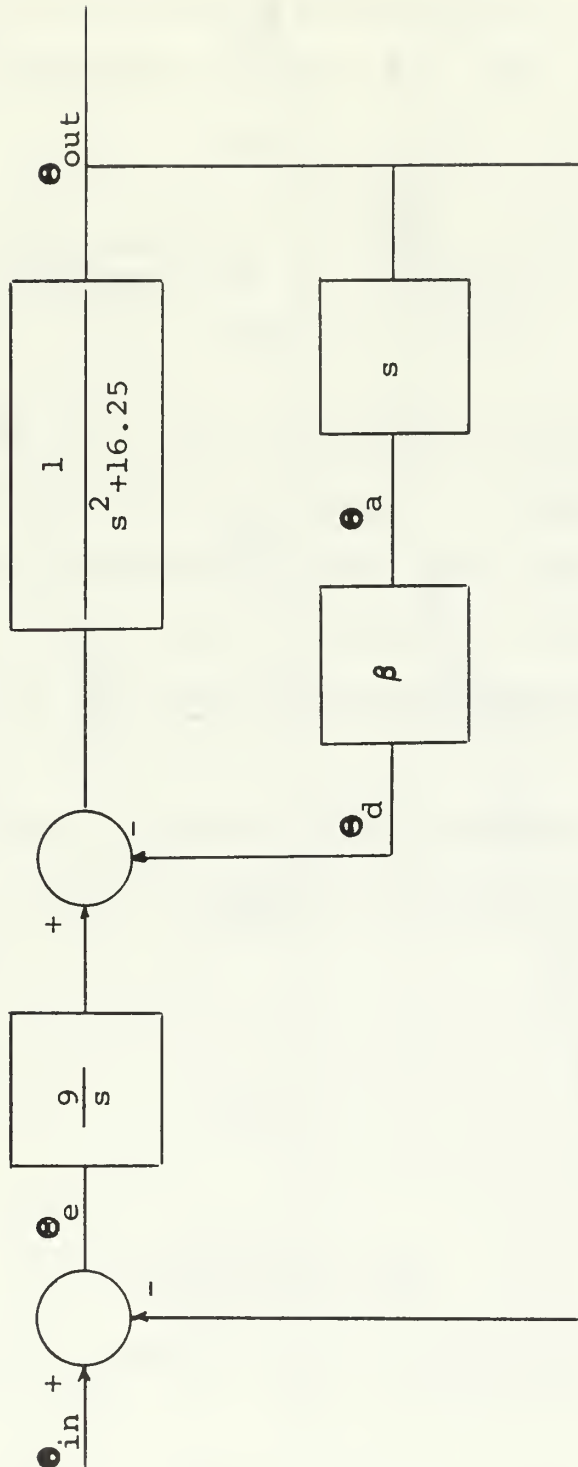
Figure 3.5 Block diagram of example #2 where β represents the variable gain of the dead zone element
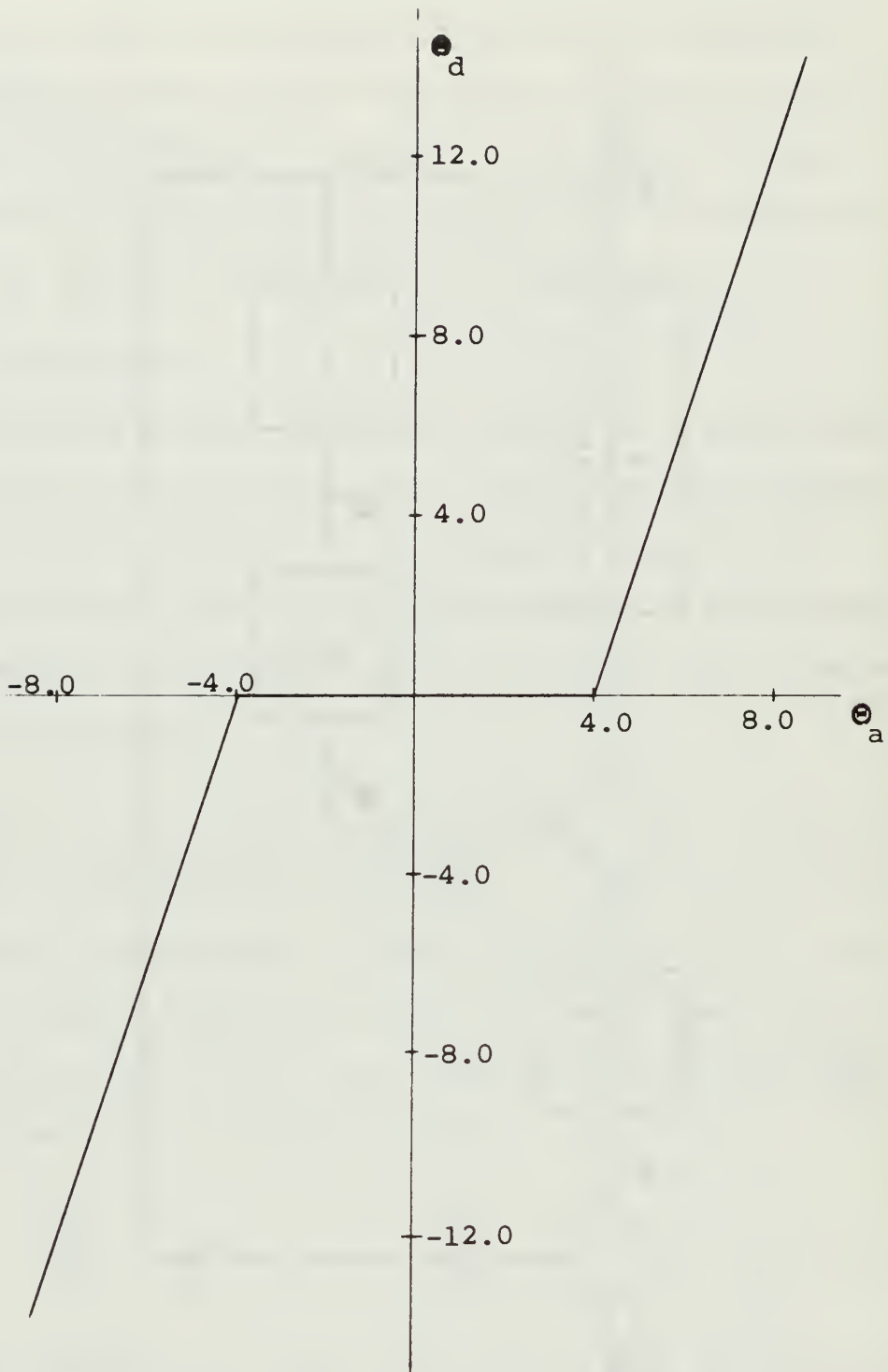
Figure 3.6 Dead zone element of example #2

Dividing the numerator and denominator of the right-hand side of equation (3.8) by Θ_{in} when $\Theta_{in} = 10.0$,

$$\beta = \frac{(3\Theta_a - 120)/\Theta_{in}}{\Theta_a/\Theta_{in}} = \frac{\dfrac{3\Theta_a}{\Theta_{in}} - 12.0}{\Theta_a/\Theta_{in}} \qquad \frac{\Theta_a}{\Theta_{in}} \geq 0.4$$

(3.9)

Equations (3.7) and (3.9) give the relation between β and $\dfrac{\Theta_e}{\Theta_{in}}$. From these two equations, a table has been tabulated of β as a function $\dfrac{\Theta_a}{\Theta_{in}}$ over the range of $\dfrac{\Theta_a}{\Theta_{in}}$ desired. This table is table (III.2) where $\dfrac{\Theta_a}{\Theta_{in}}$ is the left-hand column and β is the value read out for the corresponding value of $\dfrac{\Theta_a}{\Theta_{in}}$.

Having found the relationship between β and $\dfrac{\Theta_a}{\Theta_{in}}$, the relationship between β and ω is now found. Referring to figure (3.5),

$$\frac{\Theta_a}{\Theta_{out}} = s$$

(3.10)

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{9}{s^3 + 16.25s + 9 + \beta s^2}$$

(3.7)

Multiplying equation (3.10) by equation (3.7),

$$\frac{\Theta_a}{\Theta_{in}} = \frac{9s}{s^3 + 16.25s + 9 + \beta s^2}$$

(3.11)

61

$\dfrac{\Theta_a}{\Theta_{in}}$	β
0.0	0.0
0.05	0.0
0.10	0.0
0.15	0.0
0.20	0.0
0.25	0.0
0.30	0.0
0.35	0.0
0.40	0.0
0.45	0.333
0.50	0.600
0.55	0.819
0.60	1.00
0.65	1.16
0.70	1.28
0.75	1.40
0.80	1.50
0.85	1.59
0.90	1.67
0.95	1.74
1.00	1.80

Table III.2 β as a Function of $\dfrac{\Theta_a}{\Theta_{in}}$ for the Dead
Zone Element of Example #2

Using equation (3.11) and the PARAM-5 subprogram, a plot of β versus the magnitude of $\dfrac{\Theta_a}{\Theta_{in}}$ is obtained. This is figure (3.7) and has constant ω curves of $\omega = 1.0$, 2.0, 3.0, 3.5, 3.7, 3.8, 3.9, 4.0, 4.1, 4.2, 4.3, 4.4, 5.0, 6.0, 7.0 and 10.0 drawn on it.

The constant Θ_{in} curve for the dead zone element of this example has been drawn on figure (3.7). The coordinates for this curve are β and $\dfrac{\Theta_a}{\Theta_{in}}$, and were taken from table (III.2). The intersection of the constant Θ_{in} curve and the ω curves in figure (3.7) gives the relationship between β and ω for the system. Using this new relationship, the closed loop frequency response can now be drawn.

Equation (3.7) and the PARAM-7 subroutine are used to obtain a plot of $\dfrac{\Theta_{out}}{\Theta_{in}}$ versus ω with constant β curves over the range of $\beta = 0.0$, 0.1, 0.2, 0.4, 0.6, 0.8, 0.9, and 1.0. Using the coordinates of β and ω for the constant Θ_{in} curve on figure (3.7), the constant Θ_{in} curve is redrawn on figure (3.8). This constant Θ_{in} curve on figure (3.8) gives the closed loop frequency response of this second example with the dead zone element of figure (3.6) in the tachometer feedback path. The frequency response was checked by analog simulation and is denoted by 'x'.

D.   THIRD EXAMPLE

The third example is a fourth order system with a dead zone element in the forward path. The block diagram is figure (3.9) where α represents the variable gain of the
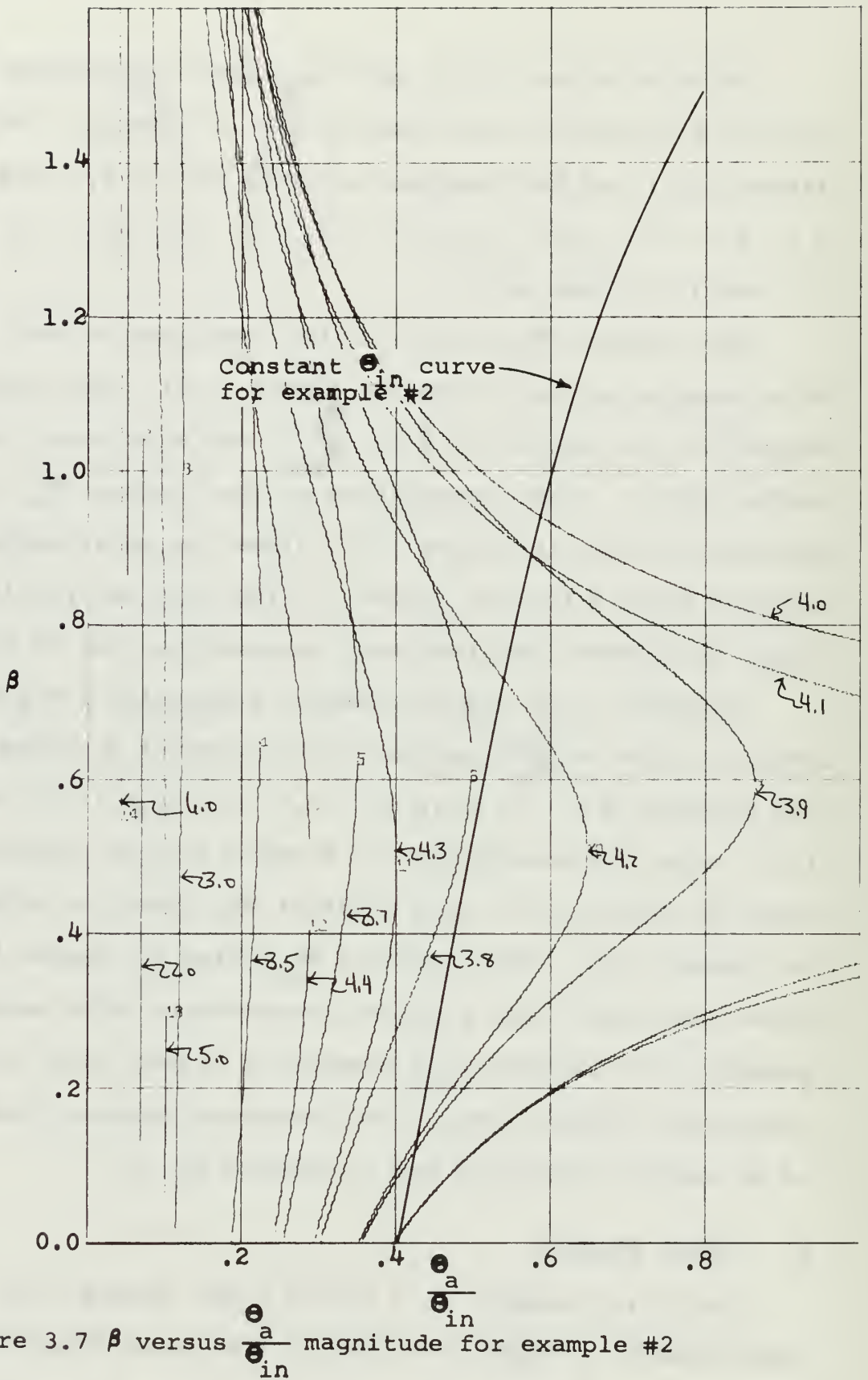
Figure 3.7 β versus $\frac{\Theta_a}{\Theta_{in}}$ magnitude for example #2

64
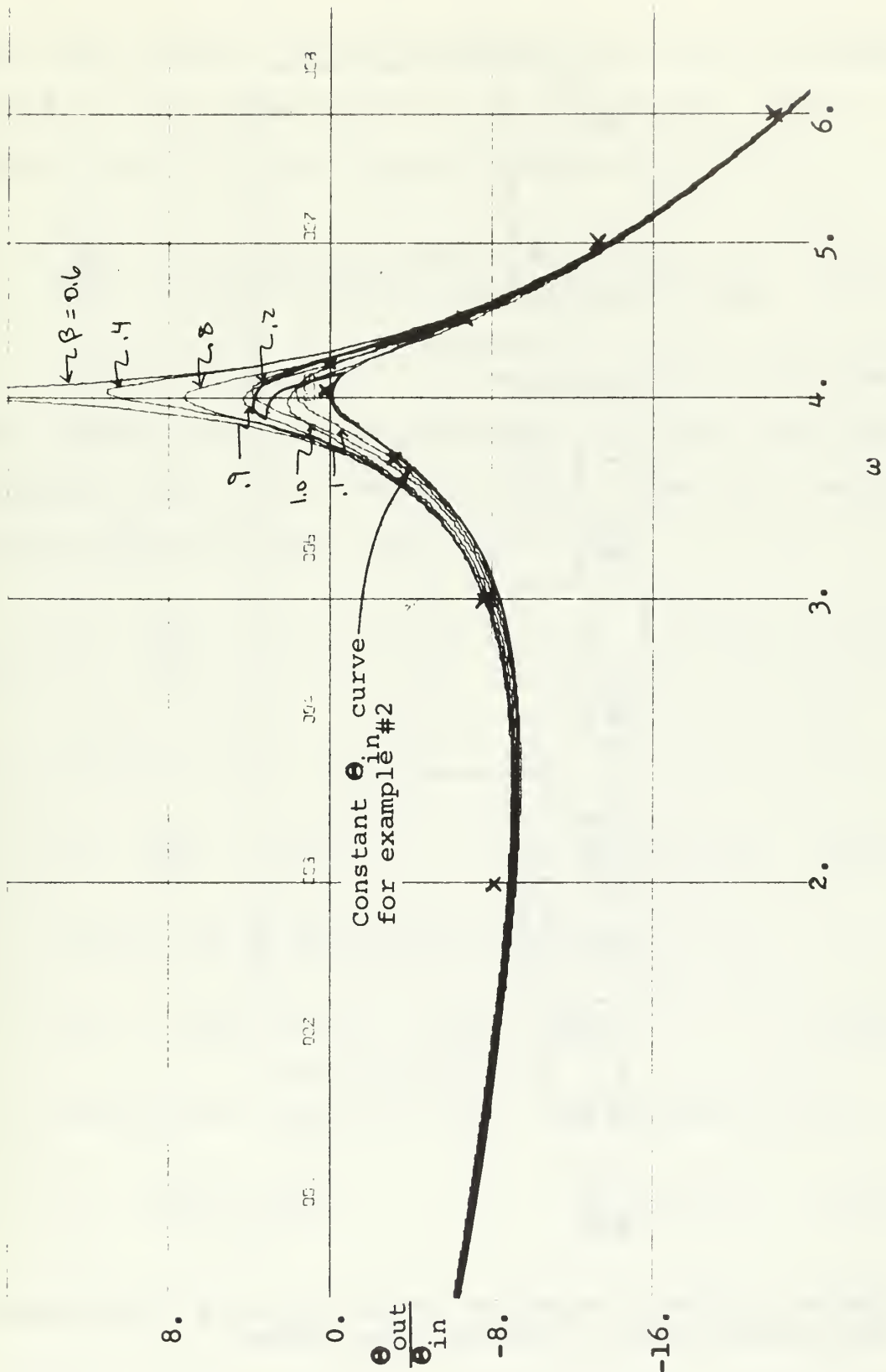
Figure 3.8 Closed loop frequency response of example #2

65
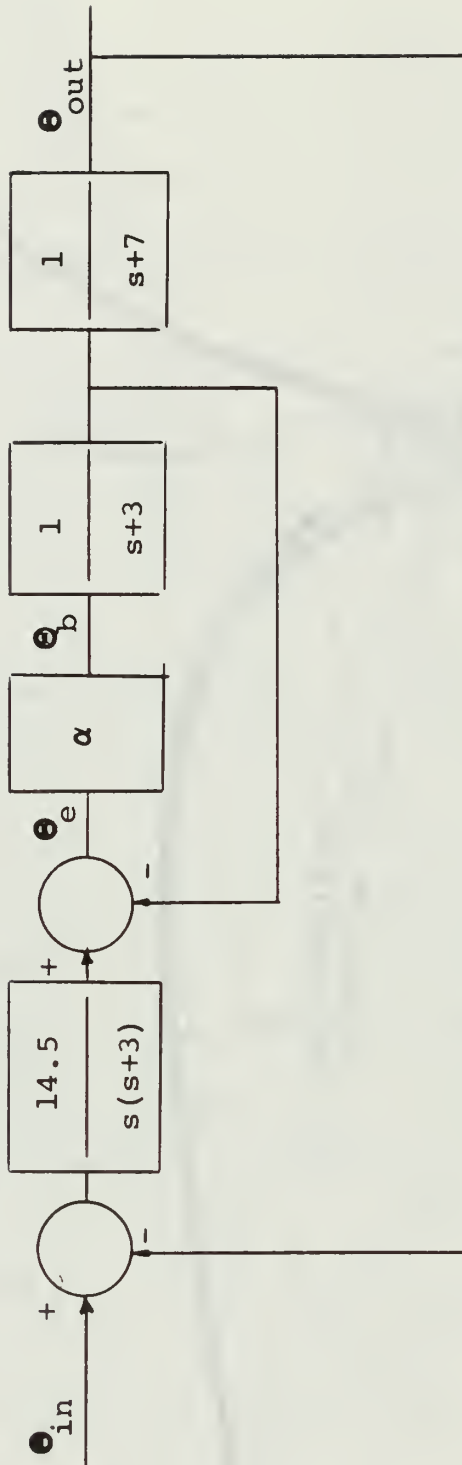
Figure 3.9 Block diagram of example #3 where α represents the variable gain of the dead zone element

dead zone element. Θ_e is the signal going into the element and Θ_b is the signal coming out of the element. From figure (3.9), the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{14.5\alpha}{s^4+13s^3+51s^2+63s+(s^3+10s^2+21s+14.5)\alpha} \qquad (3.12)$$

The dead zone element is shown in figure (3.10) where the ordinate is Θ_b and the abscissa is Θ_e. The dead space extends to $\Theta_e = 50.0$. For a Θ_e greater than 50.0, the gain of the element is two. From figure (3.9),

$$\beta = \frac{\Theta_b}{\Theta_e} \qquad (3.13)$$

For $\Theta_e \leq 50.0$,

$$\beta = \frac{0.0}{\Theta_e} = 0.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 5.0 \qquad (3.14$$

For $\Theta_e \geq 50.0$, Θ_b can be written as,

$$\Theta_b = 2(\Theta_e - 50.0) = 2\Theta_e - 100.0 \qquad (3.15)$$

Substituting equation (3.15) into equation (3.13),

$$\beta = \frac{2\Theta_e - 100.0}{\Theta_e} \qquad \Theta_e \geq 50.0 \qquad (3.16)$$

67
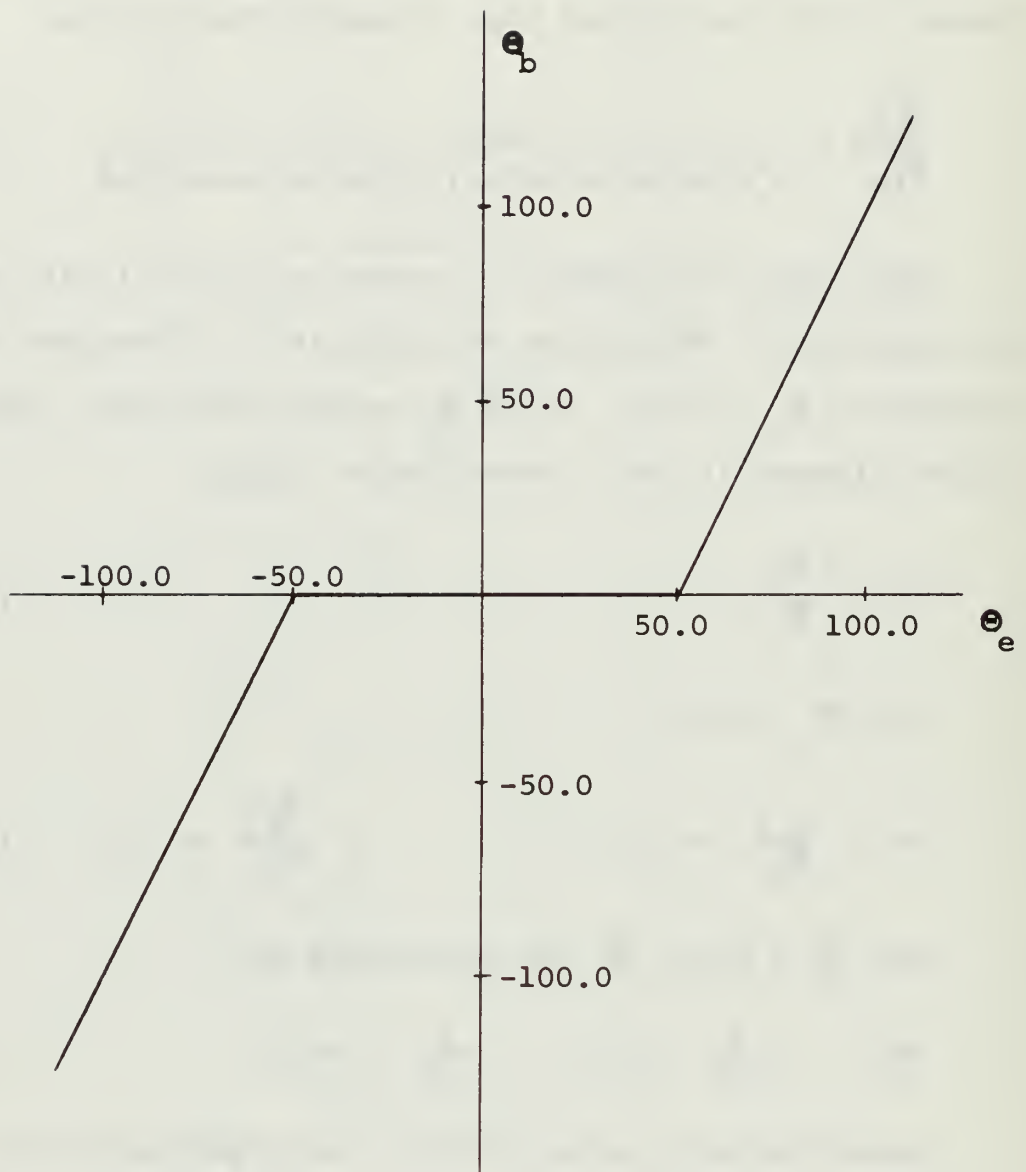
Figure 3.10 Dead zone element of example #3

Dividing the numerator and denominator of the right-hand side of equation (3.16) by Θ_{in} and realizing that when $\Theta_e \geq 50.0$, $\dfrac{\Theta_e}{\Theta_{in}} = 10.0$, equation (3.16) becomes,

$$\beta = \frac{\dfrac{2\Theta_e}{\Theta_{in}} - 10.0}{\Theta_e / \Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 5.0 \qquad (3.17)$$

From equations (3.14) and (3.16) a table of α as a function of $\dfrac{\Theta_e}{\Theta_{in}}$ is computed. This table is table (III.3) and has $\dfrac{\Theta_e}{\Theta_{in}}$ in the left-hand column and the corresponding value of α in the right-hand column. The table gives the relationship between α and $\dfrac{\Theta_e}{\Theta_{in}}$ for a constant $\Theta_{in} = 10.0$.

After finding the relationship between α and $\dfrac{\Theta_e}{\Theta_{in}}$, the relationship between α and ω is found. Referring to figure (3.9) the transfer function of $\dfrac{\Theta_e}{\Theta_{in}}$ is,

$$\frac{\Theta_e}{\Theta_{in}} = \frac{14.s^2 + 145s + 304.5}{s^4 + 13s^3 + 51s^2 + 63s + (s^3 + 10s^2 + 21s + 14.5)\alpha} \qquad (3.18)$$

Using equation (3.18) and the PARAM-5 subprogram, a plot of α versus $\dfrac{\Theta_e}{\Theta_{in}}$ is made. This plot is figure (3.11) and has ω curves of $\omega = 0.0$, 0.025, 0.05, 0.75, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0 plotted on it.

The constant Θ_{in} curve is plotted on figure (3.11) using the coordinates of α and $\dfrac{\Theta_e}{\Theta_{in}}$ tabulated in table (III.3). The intersection of this constant Θ_{in} curve

69

$\dfrac{\Theta_e}{\Theta_{in}}$	α
0.0	0.0
1.0	0.0
2.0	0.0
3.0	0.0
4.0	0.0
5.0	0.0
6.0	0.333
7.0	0.57
8.0	0.75
9.0	0.89
10.0	1.00
11.0	1.09
12.0	1.17
13.0	1.23
14.0	1.28
15.0	1.33
16.0	1.37
17.0	1.41
18.0	1.44
19.0	1.47
20.0	1.50

Table III.3 α as a Function of $\dfrac{\Theta_e}{\Theta_{in}}$ for the
Dead Zone Element of Example #3.

α

$\leftarrow \omega = 1.0$ $\leftarrow 1.1$ $\leftarrow .5$ $\leftarrow .4$ $\leftarrow .3$ $\leftarrow .2$ .1 $\rightarrow$

$\leftarrow .075$
$\leftarrow .05$
$\leftarrow .025$
$\leftarrow .01$

$\leftarrow .9$

$\leftarrow .8$

$\leftarrow .7$

1.4

1.2

1.0

.8

.6

.4

.2

0.0

Constant $\dfrac{\theta_e}{\theta_{in}}$ curve
for example #3

14 13   12   11   10   9   8   7

3.   6.   9. $\dfrac{\theta_e}{\theta_{in}}$ 12.   15.
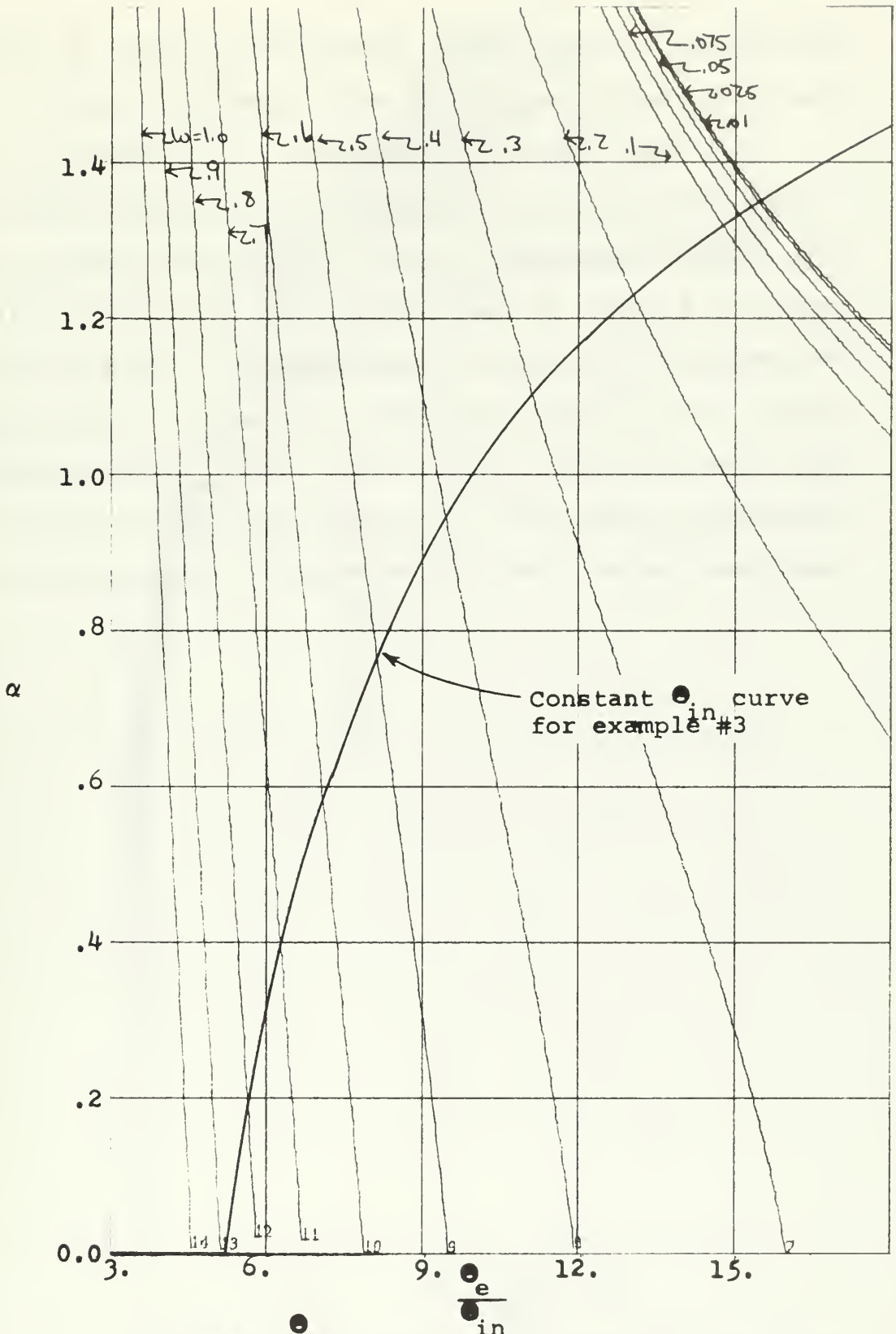
Figure 3.11 α versus $\dfrac{\theta_e}{\theta_{in}}$ magnitude for example #3

71

and the ω curves, at the associated value of α, gives
the relationship between α and ω for Θ_{in} = 10.0.

The final step in the analysis is construction of the
closed loop frequency response. Using equation (3.12) in
the PARAM-5 subprogram, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω, with
constant α lines, is made. This plot is figure (3.12).
The constant Θ_{in} curve is replotted onto figure (3.12) from
figure (3.11) using α and ω as coordinates. This constant
Θ_{in} curve in figure (3.12) is the closed loop frequency
response of example #3. An analog simulation of this ex-
ample was run and the results shown by 'x' on figure (3.12).
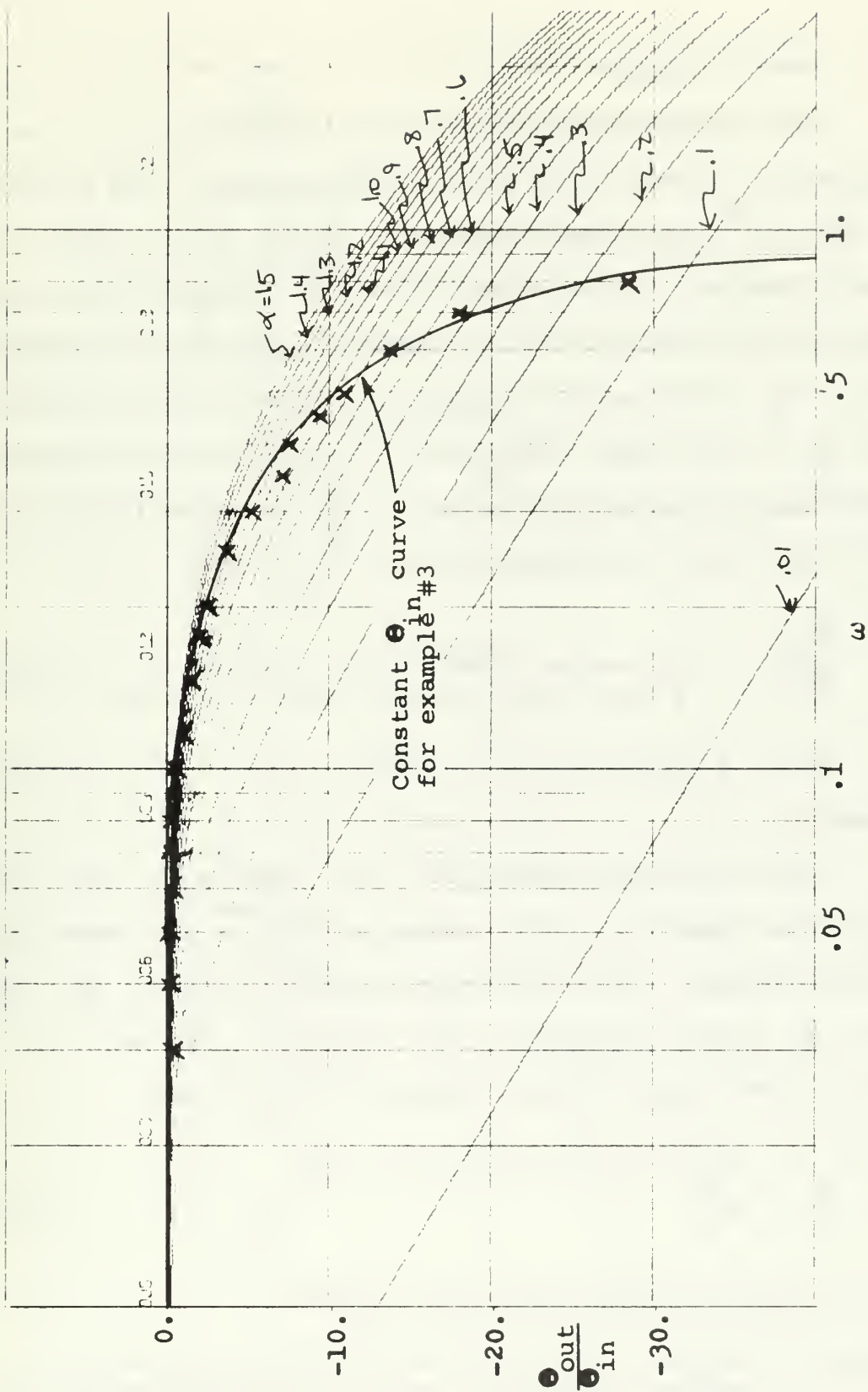
Figure 3.12 Closed loop frequency response of example #3

73

E.   FOURTH EXAMPLE

The fourth example is a fourth order system with a saturation element in a minor feedback path. The analysis is for the closed loop response. Figure (3.13) shows the block diagram of this example where β represents the variable gain of the saturation element in the minor feedback path. Θ_a is the signal going into the saturation element and Θ_d is the signal coming out of the saturation element. Referring to the block diagram of this example(figure(3.13)), the closed loop transfer function is written,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{14.5}{s^4+13s^3+51s^2+63s+14.5+\beta(s^3+10s^2+21s)} \qquad (3.19)$$

Where β represents the variable gain of the saturation element.

The saturation element for this example is figure (3.14). Θ_a is the signal into the element and Θ_d is the signal out of the element. The saturation element limits at $\Theta_a = 20.0$. For a Θ_a greater than 20.0, the output, Θ_d, is equal to 20.0. From figure (3.13), β may be written as,

$$\beta = \frac{\Theta_d}{\Theta_a} \qquad (3.20)$$

For Θ_a less than 20.0, $\Theta_a = \Theta_d$ and,

$$\beta = \frac{\Theta_a}{\Theta_a} = 1.0 \qquad\qquad \Theta_a \leq 20.0 \qquad (3.21)$$
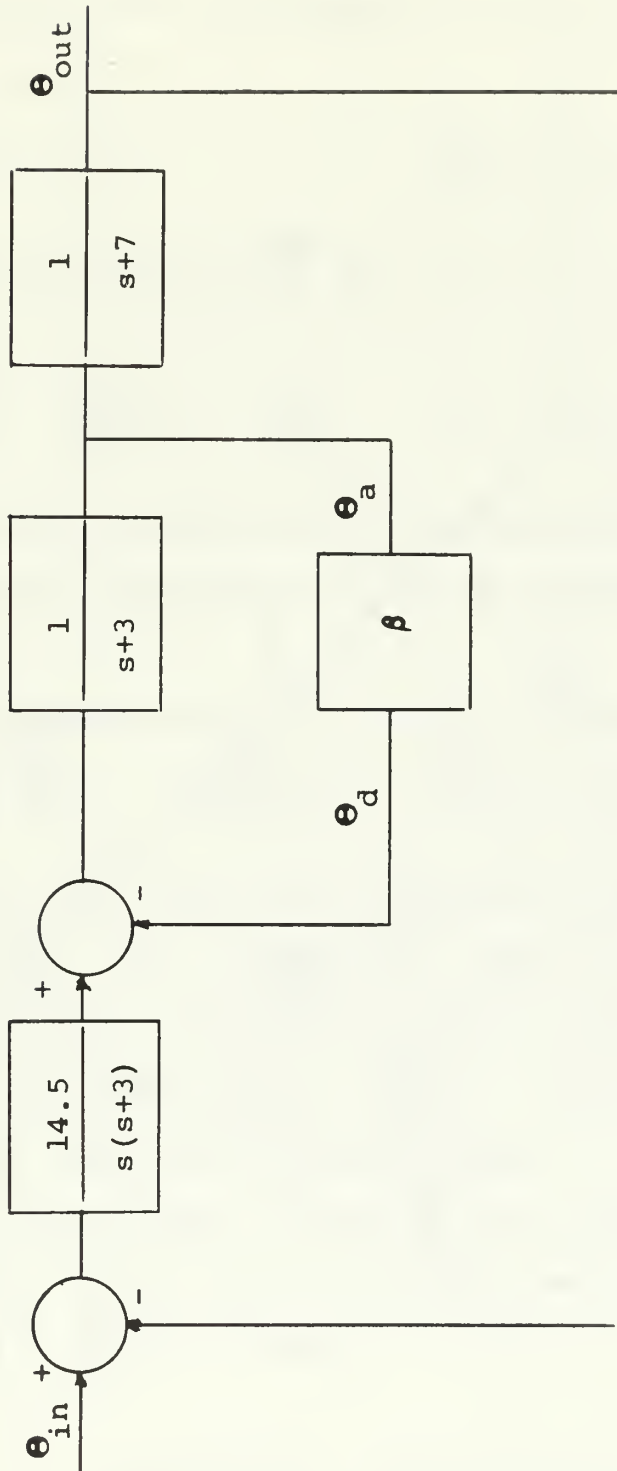
Figure 3.13 Block diagram of example #4 where β represents the variable gain of the saturation element

75
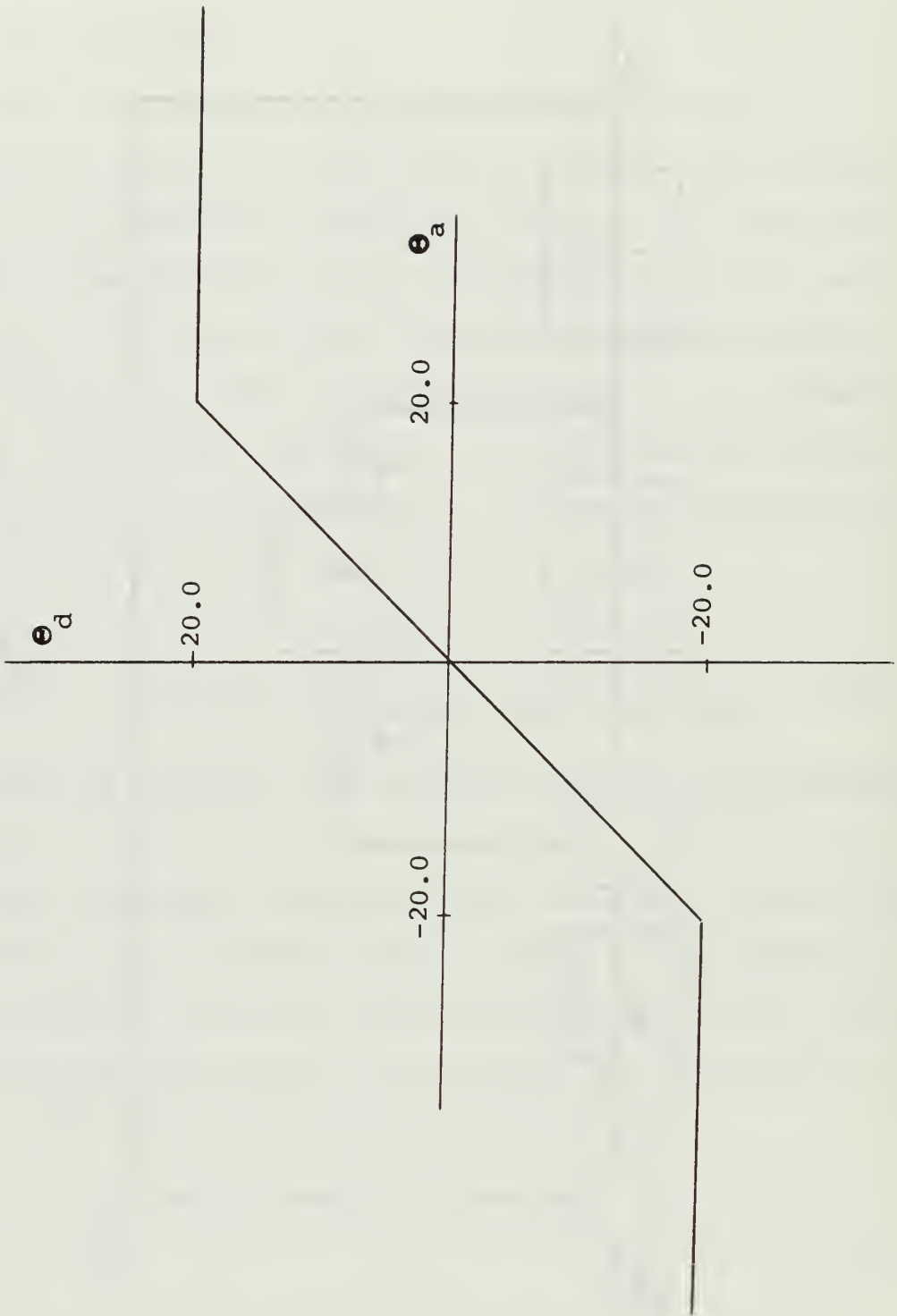
Figure 3.14 Saturation element of example #4

Dividing the limits of equation (3.21) by Θ_{in} when $\Theta_{in} = 10.0$,

$$\beta = 1.0 \qquad\qquad \frac{\Theta_a}{\Theta_{in}} \leq 2.0 \qquad (3.22)$$

For Θ_a greater than 20.0, $\Theta_d = 20.0$,

$$\beta = \frac{\Theta_d}{\Theta_a} = \frac{20.0}{\Theta_a} \qquad\qquad \Theta_a \geq 20.0 \qquad (3.23)$$

Dividing the numerator and the denominator of the right-hand side of equation (3.23) by Θ_{in} when $\Theta_{in} = 10.0$.

$$\beta = \frac{\dfrac{20.0}{\Theta_{in}}}{\Theta_a / \Theta_{in}} = \frac{2.0}{\Theta_a / \Theta_{in}} \qquad\qquad \frac{\Theta_a}{\Theta_{in}} \geq 2.0 \qquad (3.24)$$

Equations (3.22) and (3.24) define the relationship between β and $\dfrac{\Theta_e}{\Theta_{in}}$ for a constant Θ_{in} equal to 10.0. These equations are put into tabular form and are shown as table (III.4). The left-hand column is $\dfrac{\Theta_e}{\Theta_{in}}$. Given a certain $\dfrac{\Theta_e}{\Theta_{in}}$, β is read from the right-hand column.

With the relationship between $\dfrac{\Theta_e}{\Theta_{in}}$ and β established, the relationship between β and ω is now found. Referring to figure (3.13), $\dfrac{\Theta_a}{\Theta_{in}}$ is written,

$$\frac{\Theta_a}{\Theta_{in}} = \frac{14.5s+101.5}{s^4+13s^3+51s^2+63s+14.5+\beta(s^3+10s^2+21s)} \qquad (3.25)$$

77

$\dfrac{\Theta_a}{\Theta_{in}}$	β
0.0	1.0
1.0	1.0
2.0	1.0
3.0	0.667
4.0	0.500
5.0	0.400
6.0	0.333
7.0	0.286
8.0	0.250
9.0	0.222
10.0	0.200

Table III.4 β as a Function of $\dfrac{\Theta_a}{\Theta_{in}}$ for the
Saturation Element of Example #4

Using the equation (3.23) in the PARAM-5 subprogram, a plot of β versus $\frac{\Theta_a}{\Theta_{in}}$ is made. This plot is figure (3.15) and has ω curves of ω = 0.1, 0.2, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.9, and 1.0 drawn on it.

With the coordinates of β and $\frac{\Theta_a}{\Theta_{in}}$ from table (III.4), the constant Θ_{in} curve is drawn on figure (3.15). The intersection of this curve with the ω curves at the asso- ciated values of β, gives the relationship between β and ω for a constant Θ_{in} = 10.0.

Using equation (3.19) and the PARAM-7 subprogram, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω has been made. This plot is figure (3.16) and has β curves of β = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0 drawn on it. Using the coordinates of β and ω , the constant Θ_{in} curve is replot- ted onto figure (3.16). This constant Θ_{in} curve on figure (3.16) is the closed loop frequency response for example #4. The range of ω shown in figure (3.16) is only over the region in which the response deviates from a constant value of β. The response at frequencies above ω = 0.7 is the same as if the saturation element were replaced by a gain of one. The analog simulation is shown as 'x' on figure (3.16).
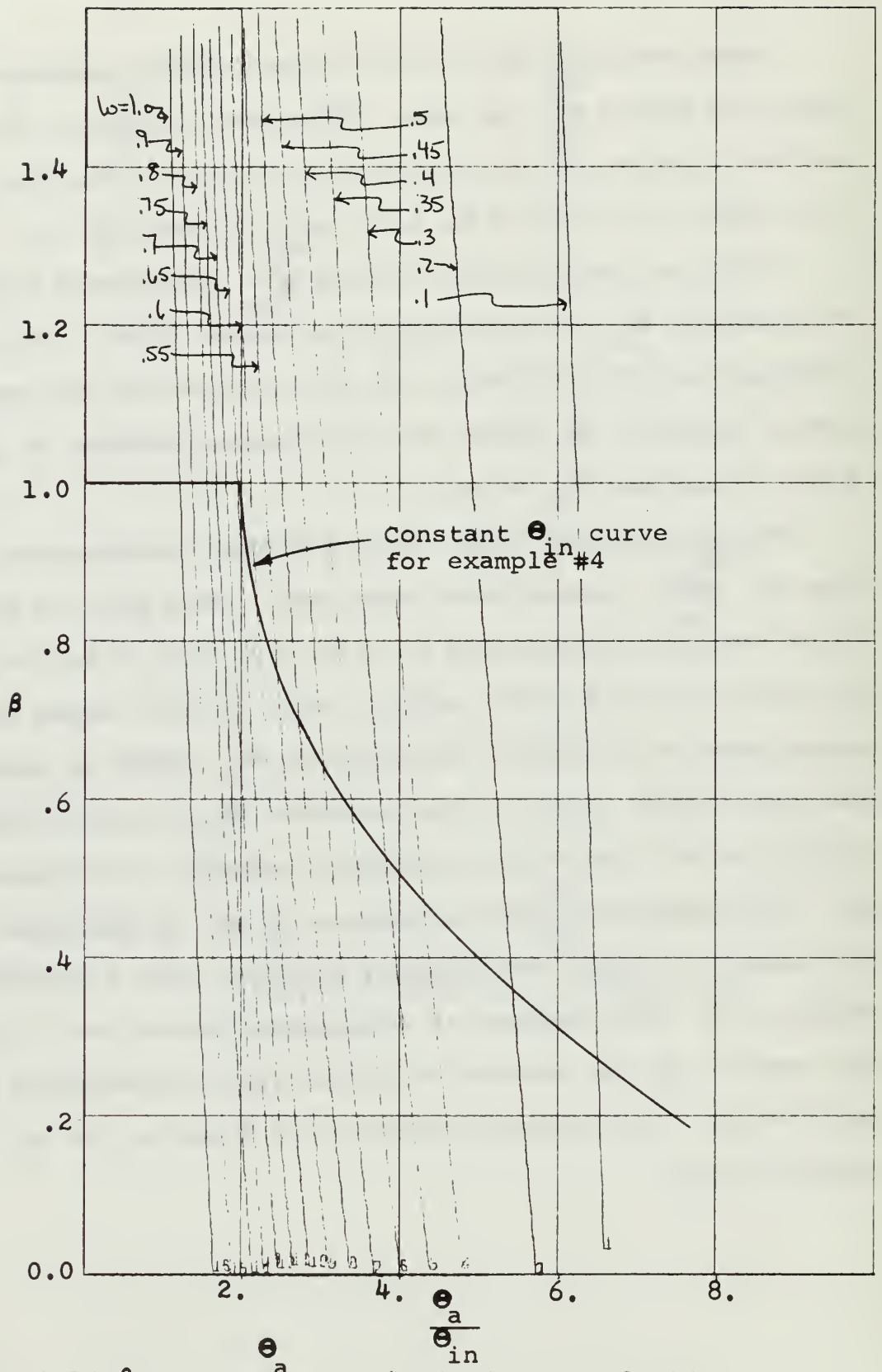
Figure 3.15 β versus $\frac{\Theta_a}{\Theta_{in}}$ magnitude for example #4
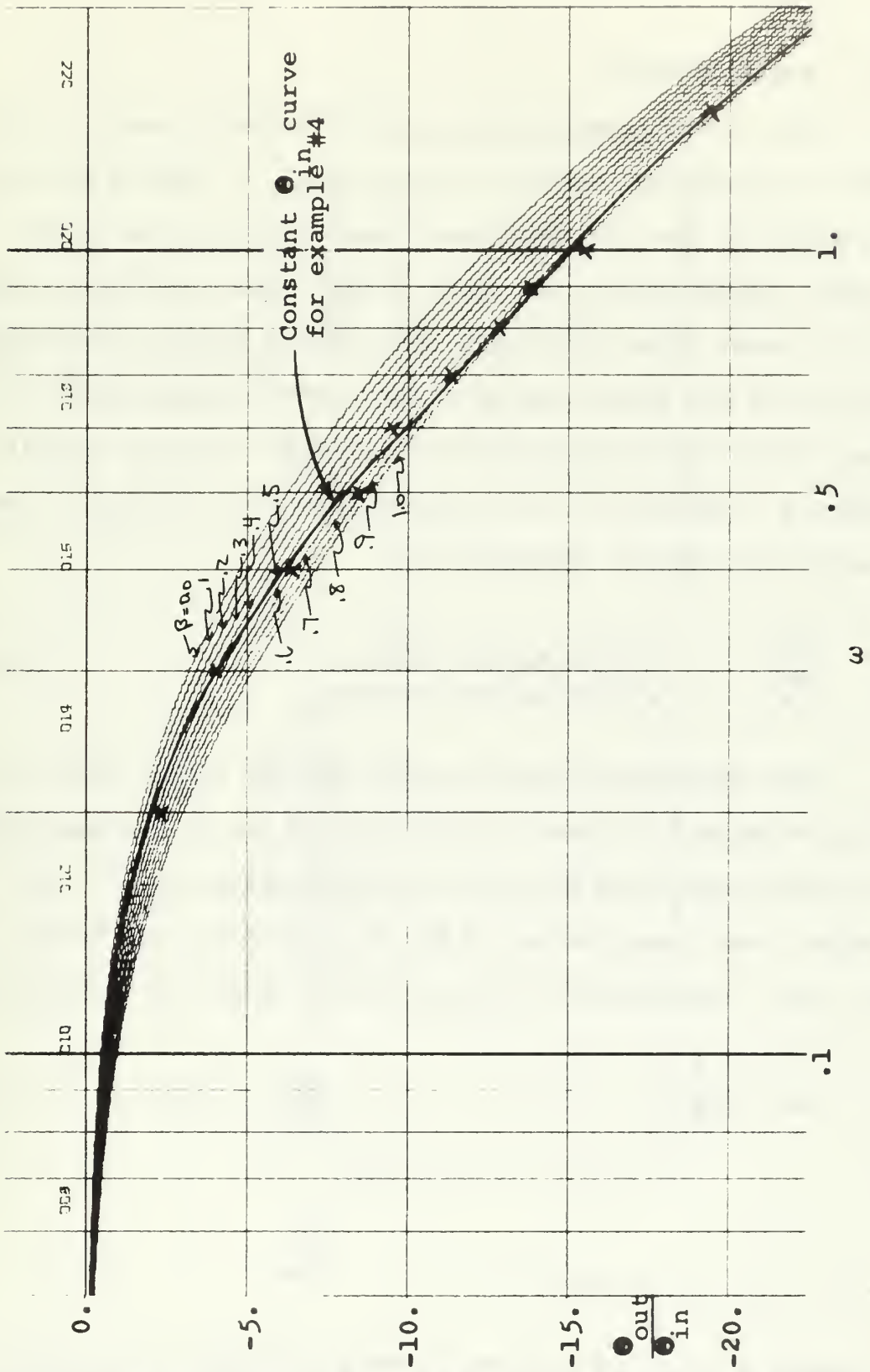
Figure 3.16 Closed loop frequency response for example #4

81

F.   FIFTH EXAMPLE

The fifth example is a type 2 feedback control system
for a reproducing contour cutting tool.[3] This system will
be analyzed for the open loop frequency response after
linear compensation has taken place. The nonlinear element
is an ideal relay which has been placed between the compen-
sator and the open loop section of the uncompensated sys-
tem. The block diagram of this system is figure (3.17)
where α represents the variable gain of the relay. The
open loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_a} = \frac{225\alpha(s^2+5.6s+7.84)}{s^2(s+5)(s^2+37.5s+350)} \tag{3.26}$$

The nonlinear characteristic of the ideal relay in
this example is figure (3.18) where Θ_e is the signal going
into the relay and Θ_b is the signal coming out of the
relay. For every value of Θ_e, Θ_b has only one value,
$\Theta_b = 40$. Referring to figure (3.17), α may be written as,

$$\alpha = \frac{\Theta_b}{\Theta_e} \qquad\qquad \Theta_e \geq 0.0 \tag{3.27}$$

---

[3]
Del Toro, V. and Parker, S.R., Principles of Control
Systems Engineering, p. 354, McGraw-Hill, 1960.

$$\frac{(10)(5)}{s^2(s+5)}$$

$$\alpha$$

$$\left(\frac{18.75(s+2.8)}{2.8(s+18.75)}\right)^2$$

θ_b

θ_e

θ_a

θ_{in}

$+$

$-$
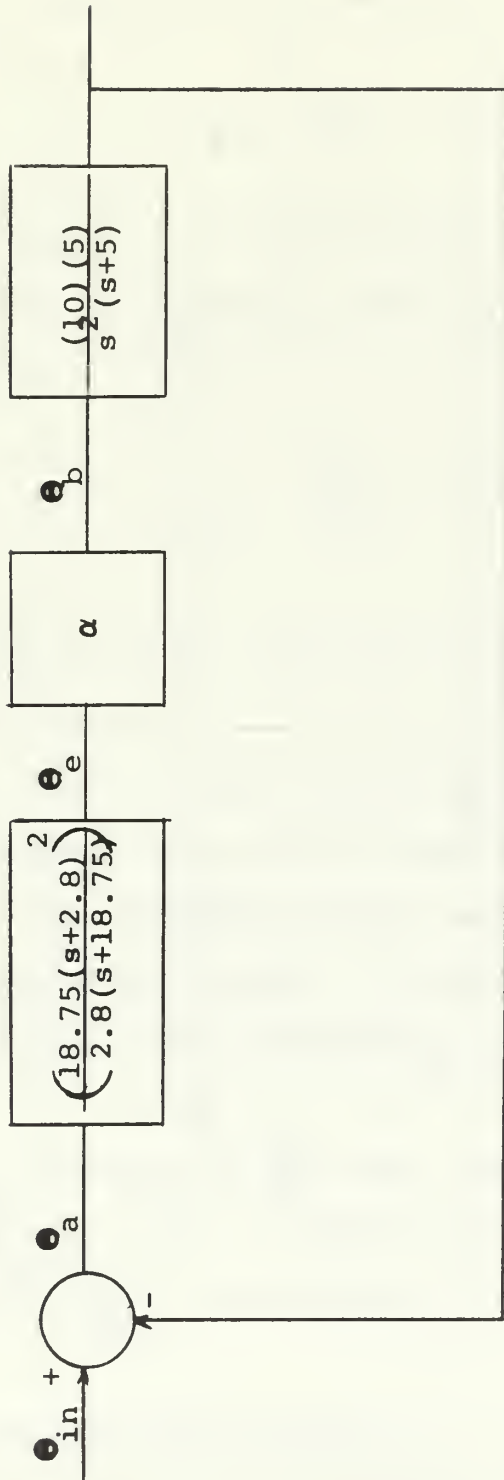
Figure 3.17 Block diagram of example #5 where α represents the variable gain of the relay
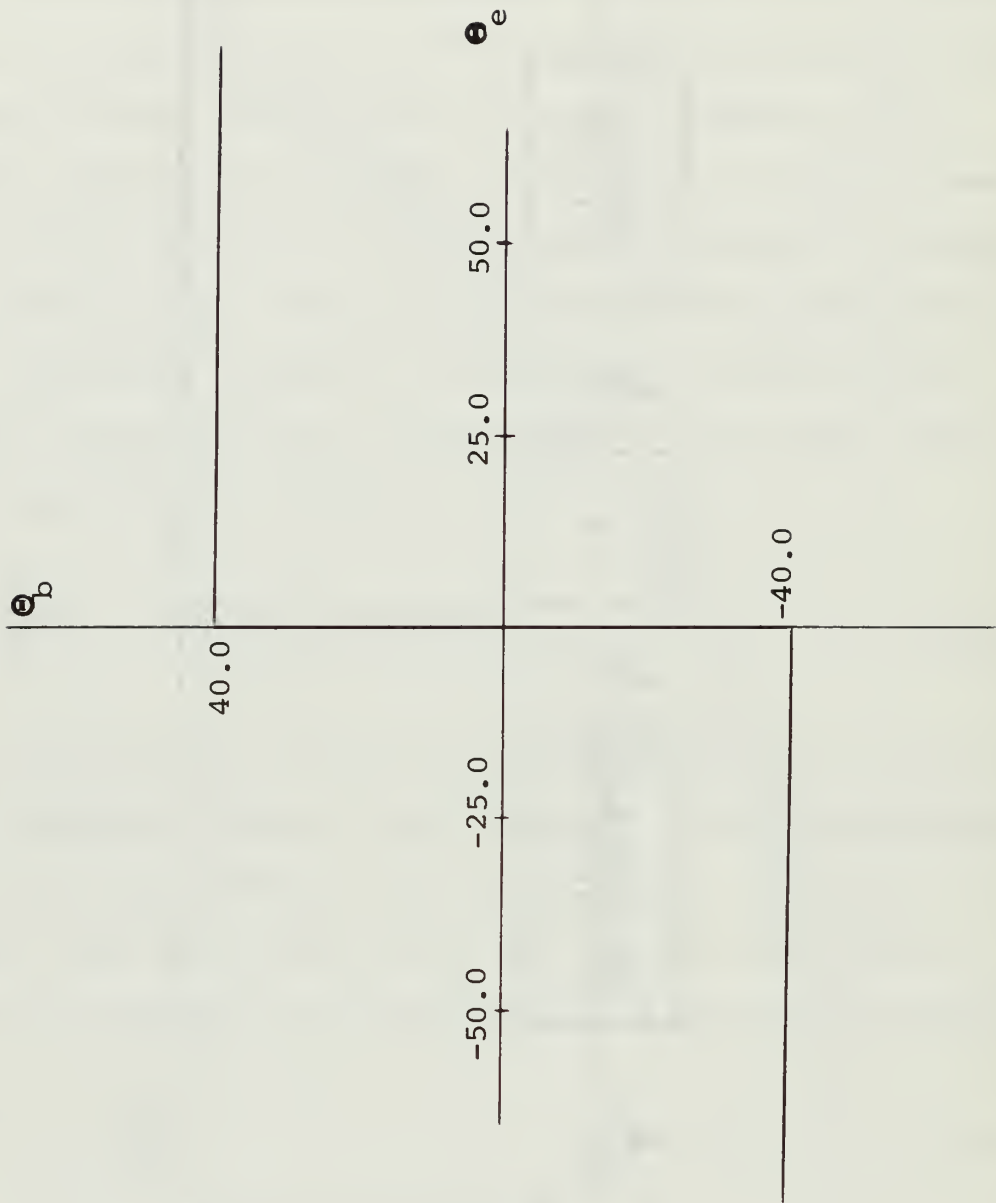
Figure 3.18 Ideal relay of example #5

For $\Theta_b = 40.$, equation (3.27) becomes,

$$\alpha = \frac{40.}{\Theta_e} \qquad\qquad \Theta_e \geq 0.0 \qquad (3.28)$$

Dividing the numerator and denominator of the right-hand side of equation (3.28) by Θ_{in} when $\Theta_{in} = 10.0$, equation (3.28) becomes,

$$\alpha = \frac{4.0}{\dfrac{\Theta_e}{\Theta_{in}}} \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.0 \qquad (3.29)$$

Equation (3.29) gives the relationship between $\dfrac{\Theta_e}{\Theta_{in}}$ and α.

From equation (3.29) a table for the relay of α as a function of $\dfrac{\Theta_e}{\Theta_{in}}$ is made up. This is table (III.5) with $\dfrac{\Theta_e}{\Theta_{in}}$ as the value in the left-hand column and the associated value of α in the right-hand column. This relationship between α and $\dfrac{\Theta_e}{\Theta_{in}}$ defines the constant Θ_{in} curve which will be used later.

Because α is a function of $\dfrac{\Theta_e}{\Theta_{in}}$ for a constant Θ_{in}, a plot of α versus $\dfrac{\Theta_e}{\Theta_{in}}$ is needed. Referring to figure (3.17), the transfer function $\dfrac{\Theta_e}{\Theta_{in}}$ is written as,

$$\frac{\Theta_e}{\Theta_{in}} = \frac{4.5s^5 + 47.7s^4 + 1610s^3 + 1768s^2}{s^5 + 42.5s^4 + 537.5s^3 + 1750s^2 + \alpha(225s^2 + 1260s + 1765)}$$

$$(3.30)$$

85

$\dfrac{\Theta_e}{\Theta_{in}}$	α
0.0	∞
0.5	8.0
1.0	4.0
1.5	2.67
2.0	2.00
2.5	1.6
3.0	1.33
3.5	1.14
4.0	1.00
4.5	0.89
5.0	0.80

Table III.5 α as a Function of $\dfrac{\Theta_e}{\Theta_{in}}$ for the Relay of Example #5

86

Using equation (3.30) and the PARAM-5 subprogram, a plot of α versus $\dfrac{\Theta_e}{\Theta_{in}}$ is made. This plot is figure (3.19) and has drawn on it constant ω curves of $\omega = 0.25$, 0.5, 0.75, 1.0, 1.25, 1.75, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 20.0 and 40.0.

The constant Θ_{in} curve is drawn on figure (3.19) using the coordinates of α and $\dfrac{\Theta_e}{\Theta_{in}}$ from table (III.5). The intersection of this constant Θ_{in} curve and the ω curves in figure (3.19) gives the new relationship between α and ω for a $\Theta_{in} = 10.0$. With this final relationship, the open loop frequency response is drawn for the system with the relay.

From equation (3.26) and the PARAM-7 subprogram, a plot of $\dfrac{\Theta_{out}}{\Theta_a}$ (magnitude - dB and phase) versus ω is made. On this plot α curves are drawn of $\alpha = 0.7$, 0.85, 1.0, 1.1, 1.25, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 6.0, 7.0, 8.0 and 15.0. The $\dfrac{\Theta_{out}}{\Theta_{in}}$ versus ω plot is figure (3.20). Note that for different values of α, the phase response is the same. This indicates that the nonlinear element does not change the system from its original linear phase response. The $\alpha = 1.0$ curve is the linear open loop magnitude response. Using the coordinates of α and ω, the constant Θ_{in} curve on figure (3.19) is replotted onto figure (3.20). The constant Θ_{in} curves on figure (3.20) are the open loop magnitude and phase response of example #5 with an ideal relay.

Figure 3.19 α versus $\dfrac{\Theta_e}{\Theta_{in}}$ for example #5

88
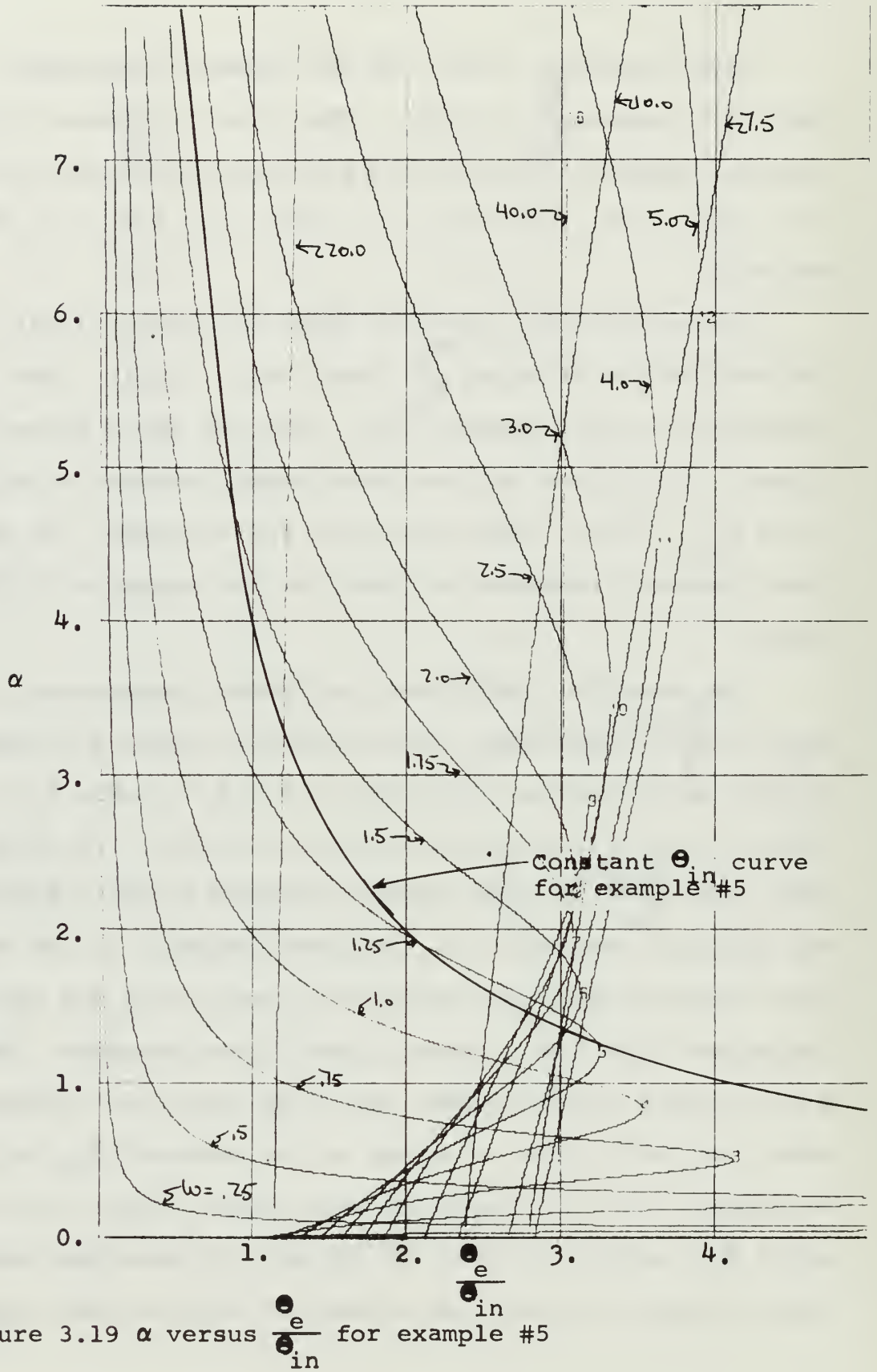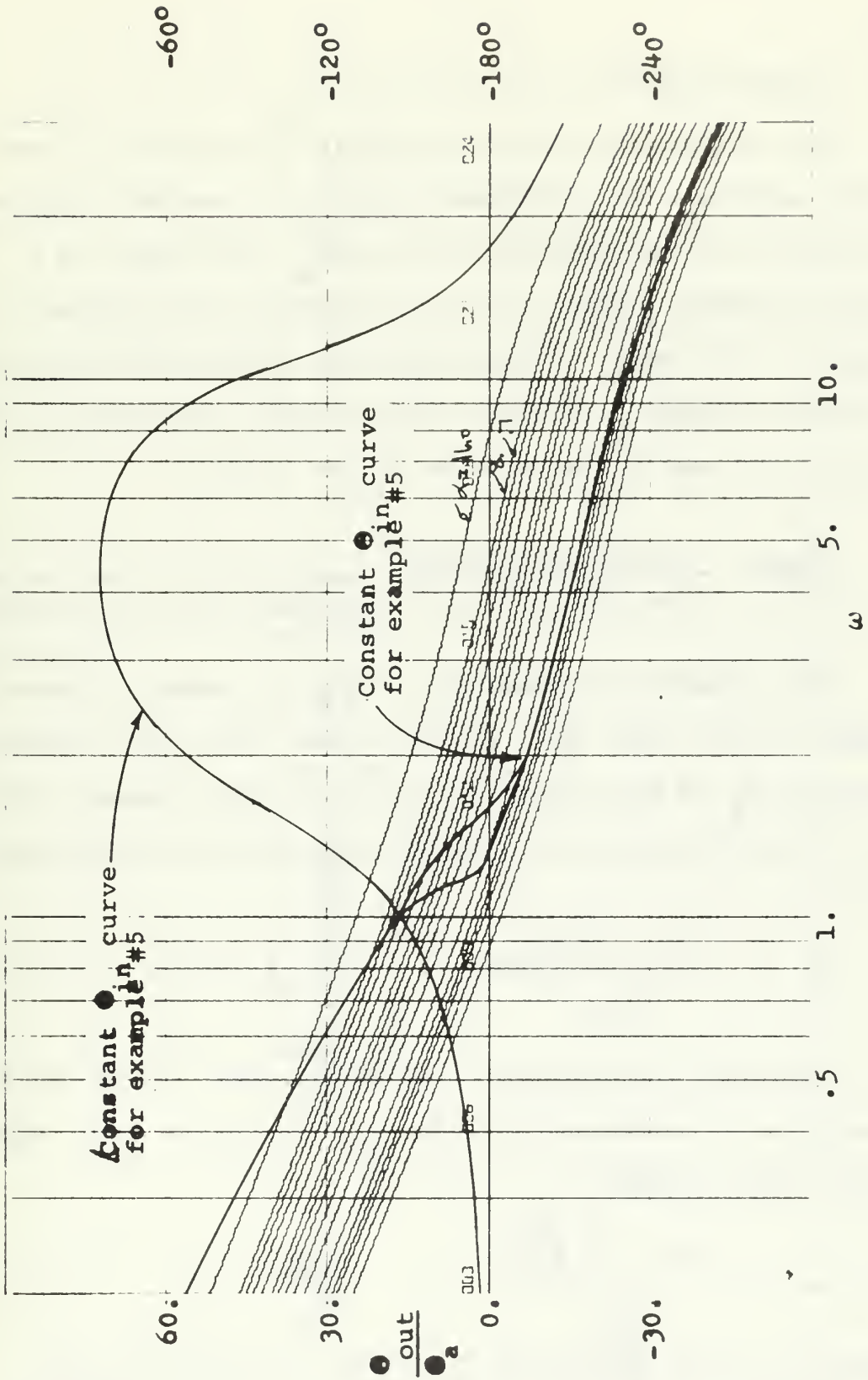
Figure 3.20 Open loop frequency response of example #5

89

G. SIXTH EXAMPLE

The system for the sixth example is similar to the fifth example. The nonlinear element is coulomb friction plus stiction which converts velocity into torque in a minor feedback loop. The block diagram of the system is figure (3.21) where α represents the coulomb friction plus stiction element. The open loop transfer function is equation (3.31) and is taken from figure (3.21).

$$\frac{\Theta_{out}}{\Theta_a} = \frac{225(s^2+5.6s+7.84)}{s^5+42.5s^4+537.5s^3+1750s^2+\alpha(s^4+42.5s^3+537.5s^2+1750s)}$$

$$(3.31)$$

The coulomb friction with stiction element is shown in figure (3.22) where $\dot{\Theta}_{out}$ is the signal going into the element and T_1 is the signal coming out of the element. For $\dot{\Theta}_{out}$ less than 2.0, the variable gain can be written as,

$$\alpha = \frac{4.17 - \frac{1}{3}\dot{\Theta}_{out}}{\dot{\Theta}_{out}} \qquad \dot{\Theta}_{out} \leq 2.0 \qquad (3.32)$$

Dividing the numerator and denominator of the right-hand side of equation (3.32) by Θ_{in} when $\Theta_{in} = 10.0$, equation (3.32) becomes,

$$\alpha = \frac{.417 - \frac{1}{3}\frac{\dot{\Theta}_{out}}{\Theta_{in}}}{\frac{\dot{\Theta}_{out}}{\Theta_{in}}} \qquad \frac{\dot{\Theta}_{out}}{\Theta_{in}} \leq 0.2 \qquad (3.33)$$
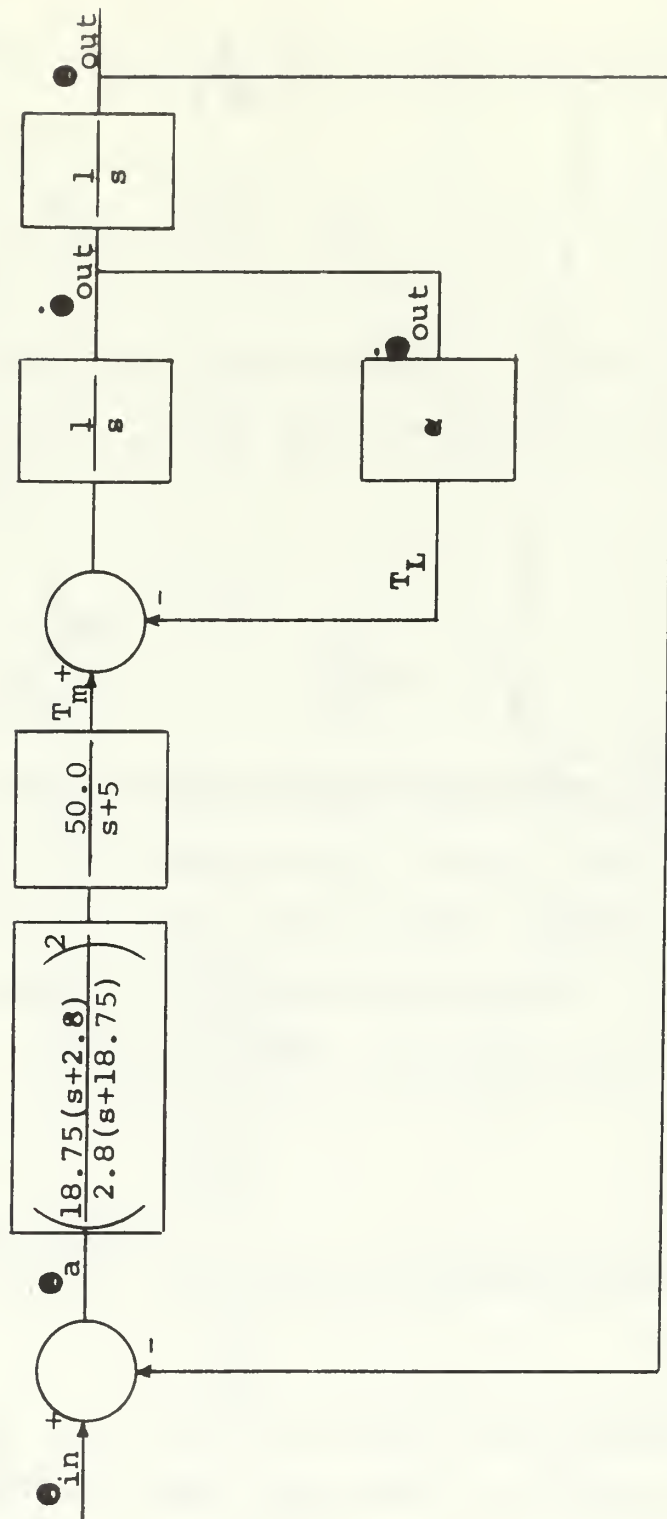
Figure 3.21 Block diagram of example #6 where α represents the variable gain of the coulomb friction plus stiction element

91
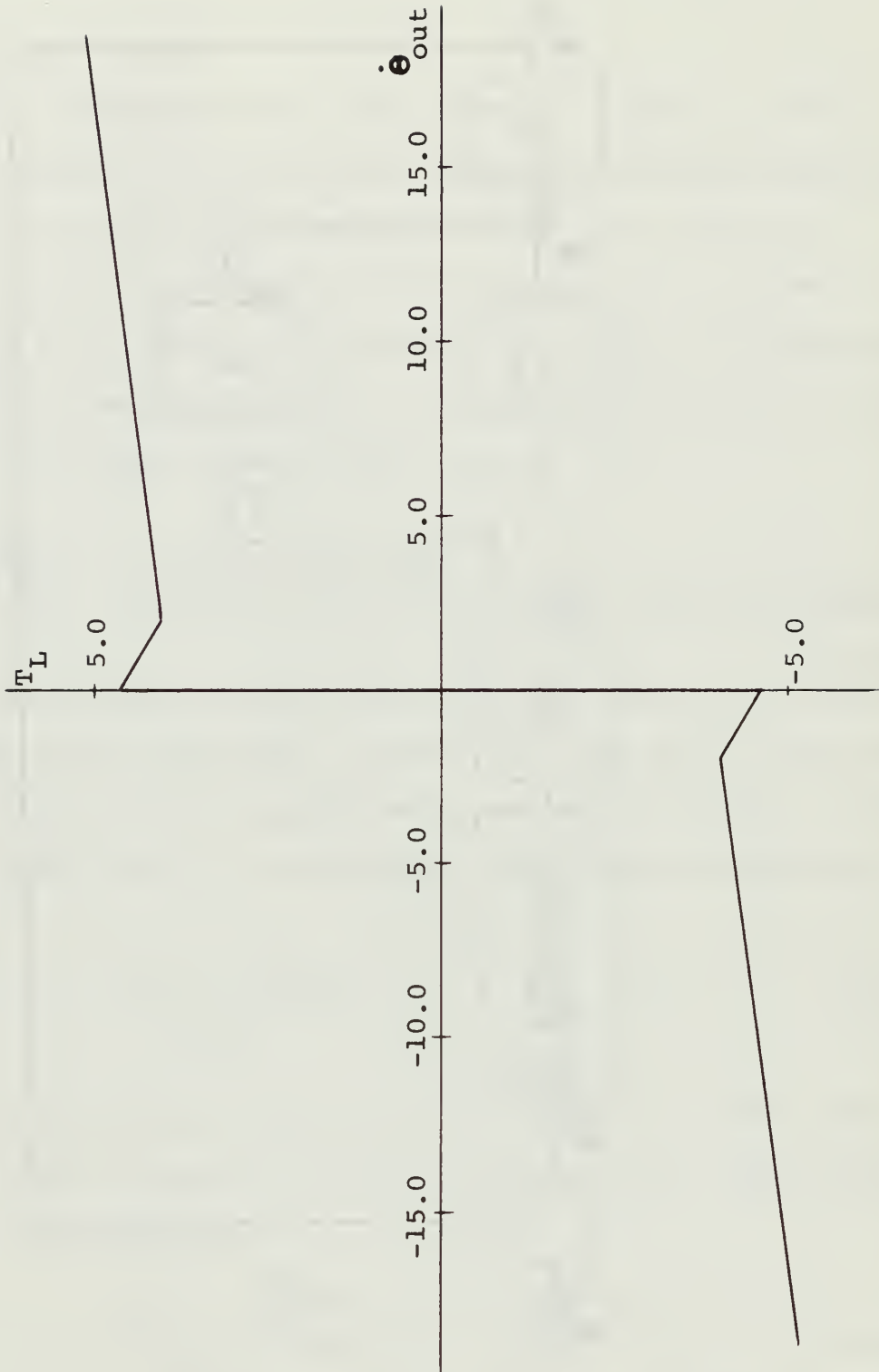
Figure 3.22 Coulomb friction plus stiction element of example #6

For Θ_{out} greater than 2.0, the variable gain can be written as,

$$\alpha = \frac{3.3 + .1\ \dot{\Theta}_{out}}{\Theta_{out}} \qquad \dot{\Theta}_{out} \geq 2.0 \qquad (3.34)$$

Dividing the numerator and denominator of the right-hand side of equation (3.34) by Θ_{in} when $\Theta_{in} = 10.0$, equation (3.34) becomes,

$$\alpha = \frac{.33 + .1\ \dfrac{\dot{\Theta}_{out}}{\Theta_{in}}}{\Theta_{out}/\Theta_{in}} \qquad \frac{\dot{\Theta}_{out}}{\Theta_{in}} \geq 0.20 \qquad (3.35)$$

Equations (3.33) and (3.34) define the relationship between α and $\dfrac{\Theta_{out}}{\Theta_{in}}$ for a constant $\Theta_{in} = 10.0.$ This relationship is tabulated in table (III.6) where $\dfrac{\dot{\Theta}_{out}}{\Theta_{in}}$ is the left-hand column and α is the right-hand column.

The transfer function of $\dfrac{\dot{\Theta}_{out}}{\Theta_{in}}$ is written from looking at figure (3.21).

$$\frac{\dot{\Theta}_{out}}{\Theta_{in}} = \frac{225s^3 + 1260s^2 + 1764s}{s^5 + 42.5s^4 + 537.5s^3 + 1975s^2 + 1260s + 1764 + \alpha(s^4 + 42.5s^3 + 537.5s^2 + 1750s)}$$

$$(3.36)$$

Using equation (3.36) and the PARAM-5 subprogram, a plot of α versus $\dfrac{\dot{\Theta}_{out}}{\Theta_{in}}$ is made. This plot is figure (3.23) and has constant ω curves between $\omega = 0.01$ and $\omega = 10.0$ drawn on it. Using the relationship between α and $\dfrac{\dot{\Theta}_{out}}{\Theta_{in}}$ from

$\dfrac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$	α
.00	∞
.05	8.0
.10	3.84
.15	2.45
.20	1.75
.3	1.2
.4	.95
.5	.76
.6	.65
.7	.57
.8	.51
.9	.466
1.0	.43
1.1	.40
1.2	.375
1.3	.354
1.4	.335
1.5	.32

Table III.6 α as a Function of $\dfrac{\dot{\Theta}_{out}}{\dot{\Theta}_{in}}$ for Coulomb

Friction Plus Stiction in Example #6

Figure 3.23 α versus $\dfrac{\dot{\theta}_{out}}{\dot{\theta}_{in}}$ magnitude for example #6

95
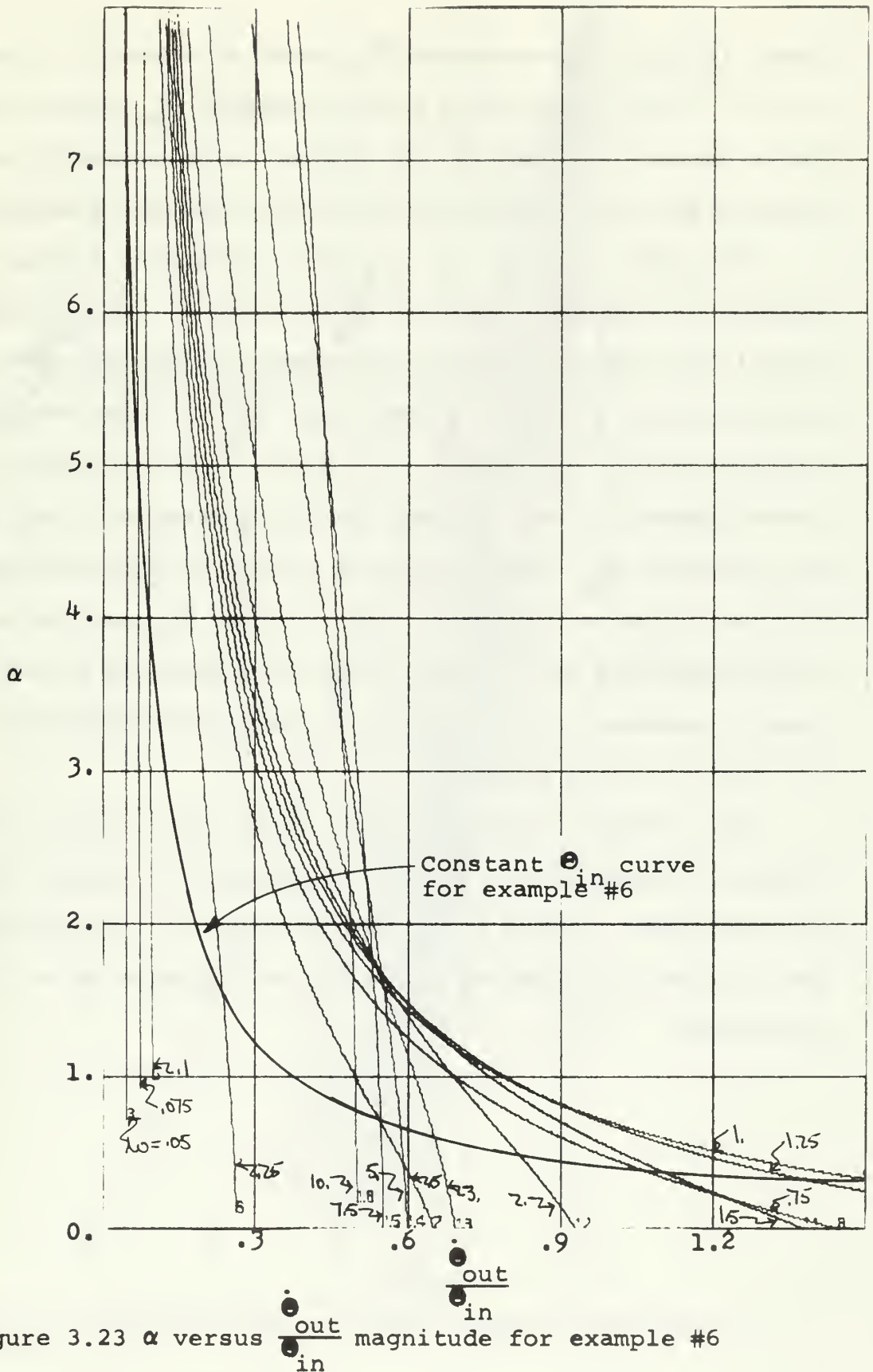
table (III.6), the constant Θ_{in} curve is drawn on figure (3.23). The intersection of the constant Θ_{in} curve with the ω curves in figure (3.23) gives the relationship between α and ω for the coulomb friction plus stiction element.

The final step in the open loop frequency response analysis is making a plot of $\dfrac{\Theta_{out}}{\Theta_a}$ versus ω. Using equation (3.31) and the PARAM-7 subprogram, plots of $\dfrac{\Theta_{out}}{\Theta_a}$ magnitude versus ω (figure (3.24)) and $\dfrac{\Theta_{out}}{\Theta_a}$ - phase versus ω (figure (3.25)) are made. Both plots have constant α curves drawn on them. Using the coordinates of α and ω, the constant Θ_{in} curve in figure (3.23) is replotted onto figures (3.24) and (3.25). The constant Θ_{in} curves on these two plots are the open loop magnitude and phase frequency response of this example. The linear response ($\alpha = 0.0$) is also shown.

For a better analysis, the closed loop response is shown in figure (3.26) for the magnitude and figure (3.27) for the phase. These plots were obtained in the same manner as previous examples. The linear response ($\alpha = 0.0$) is also shown.
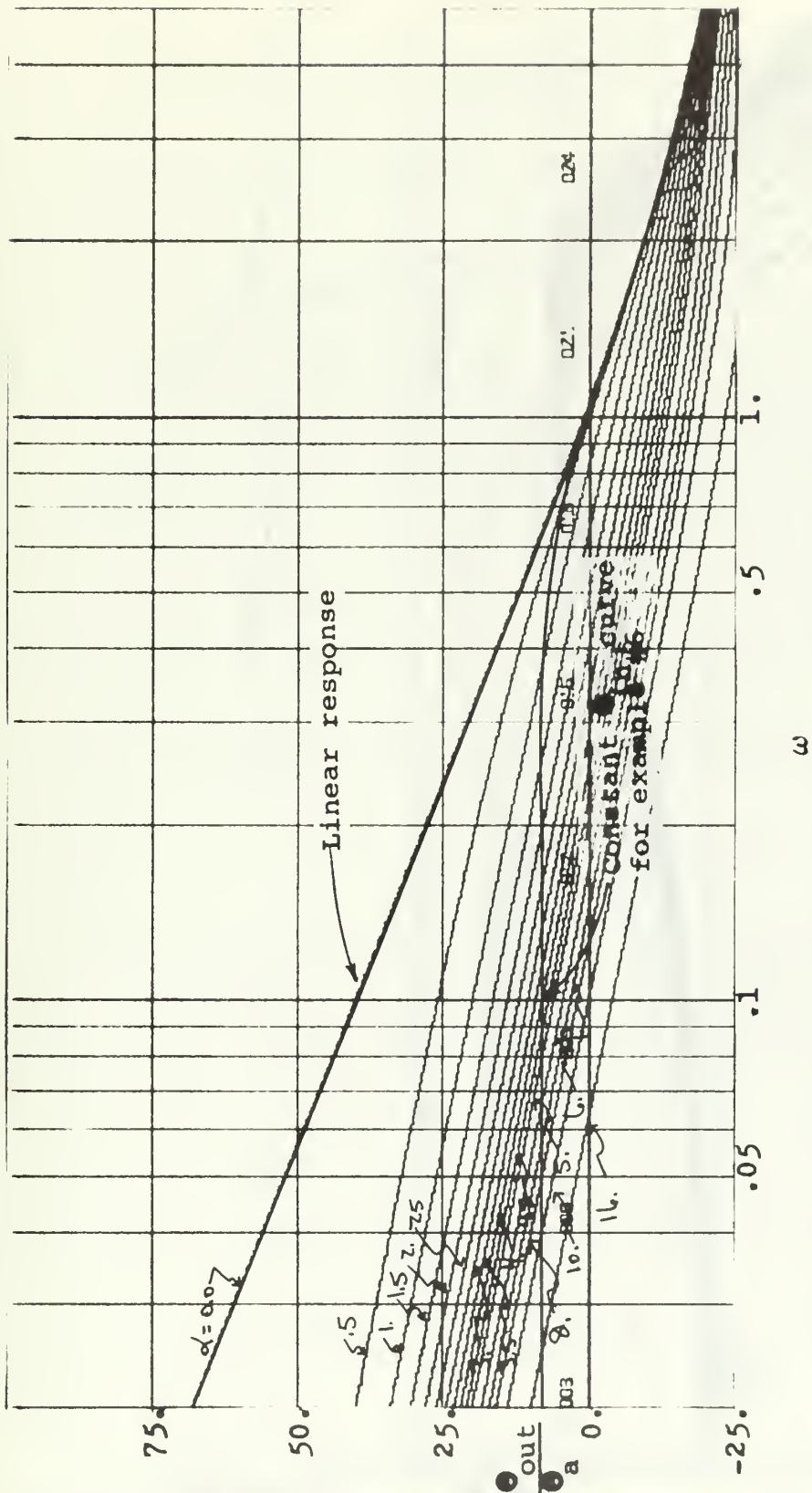
Figure 3.24 Open loop magnitude frequency response of example #6
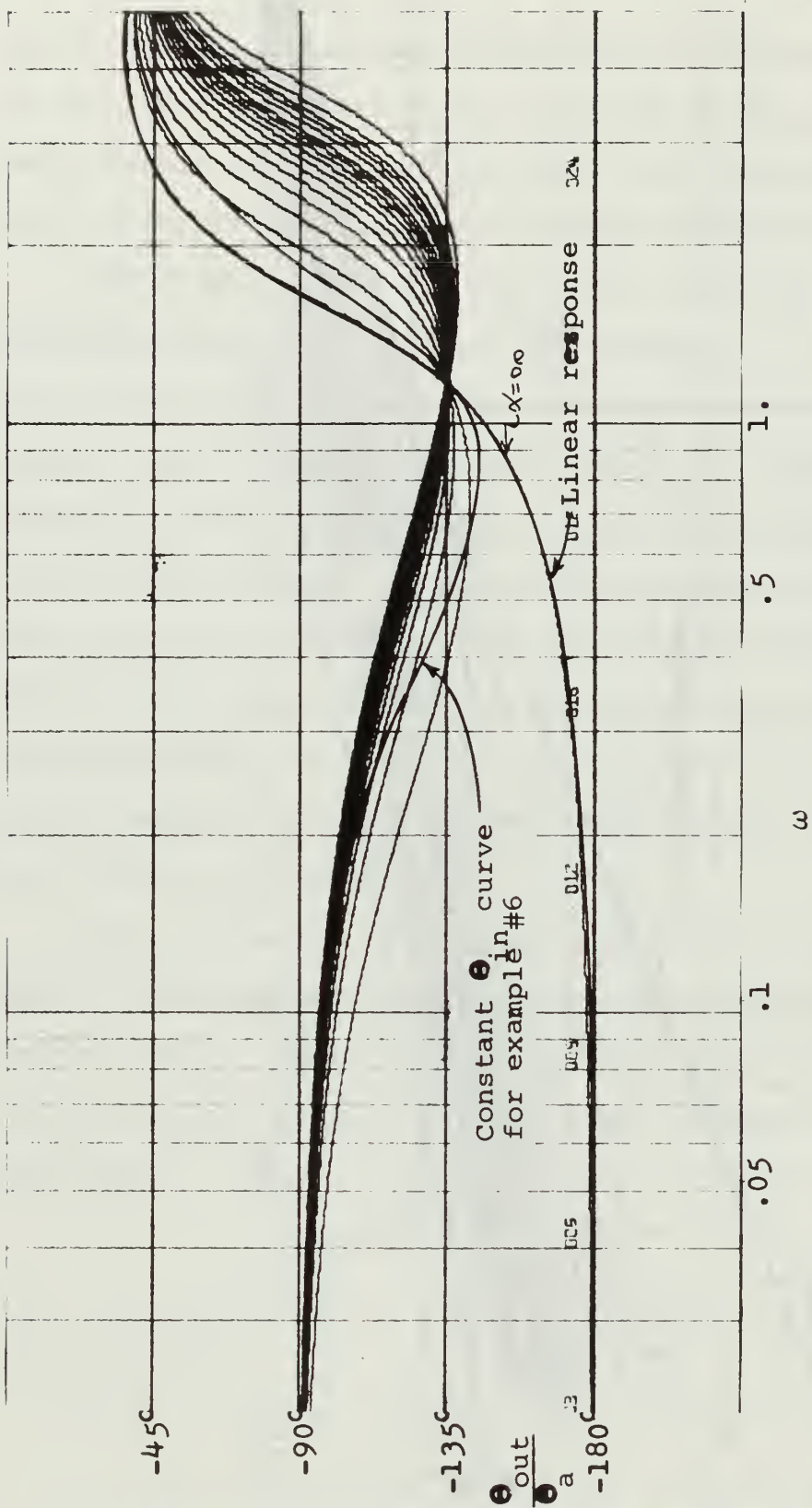
Figure 3.25 Open loop phase frequency response of example #6

98
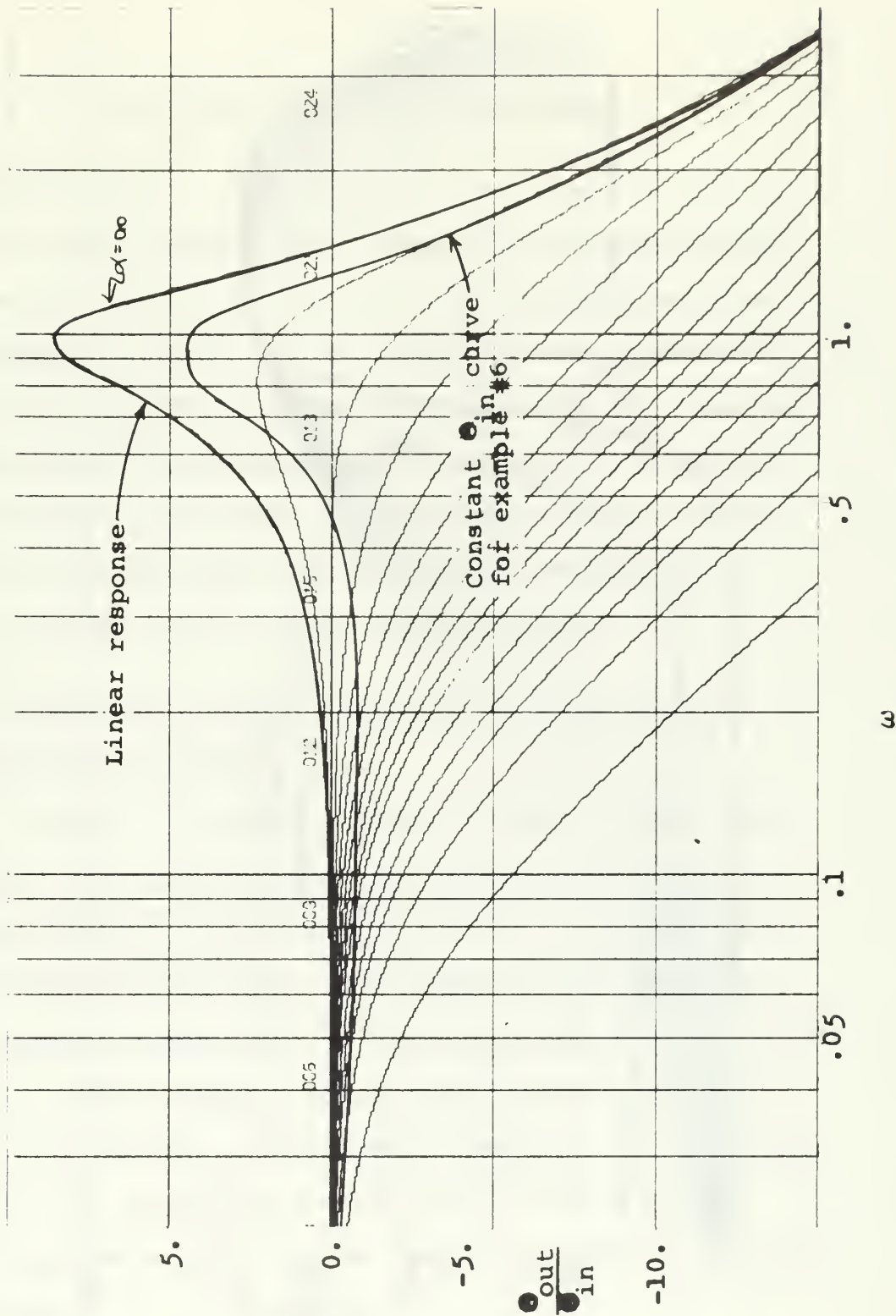
Figure 3.26 Closed loop magnitude frequency response of example #6

99
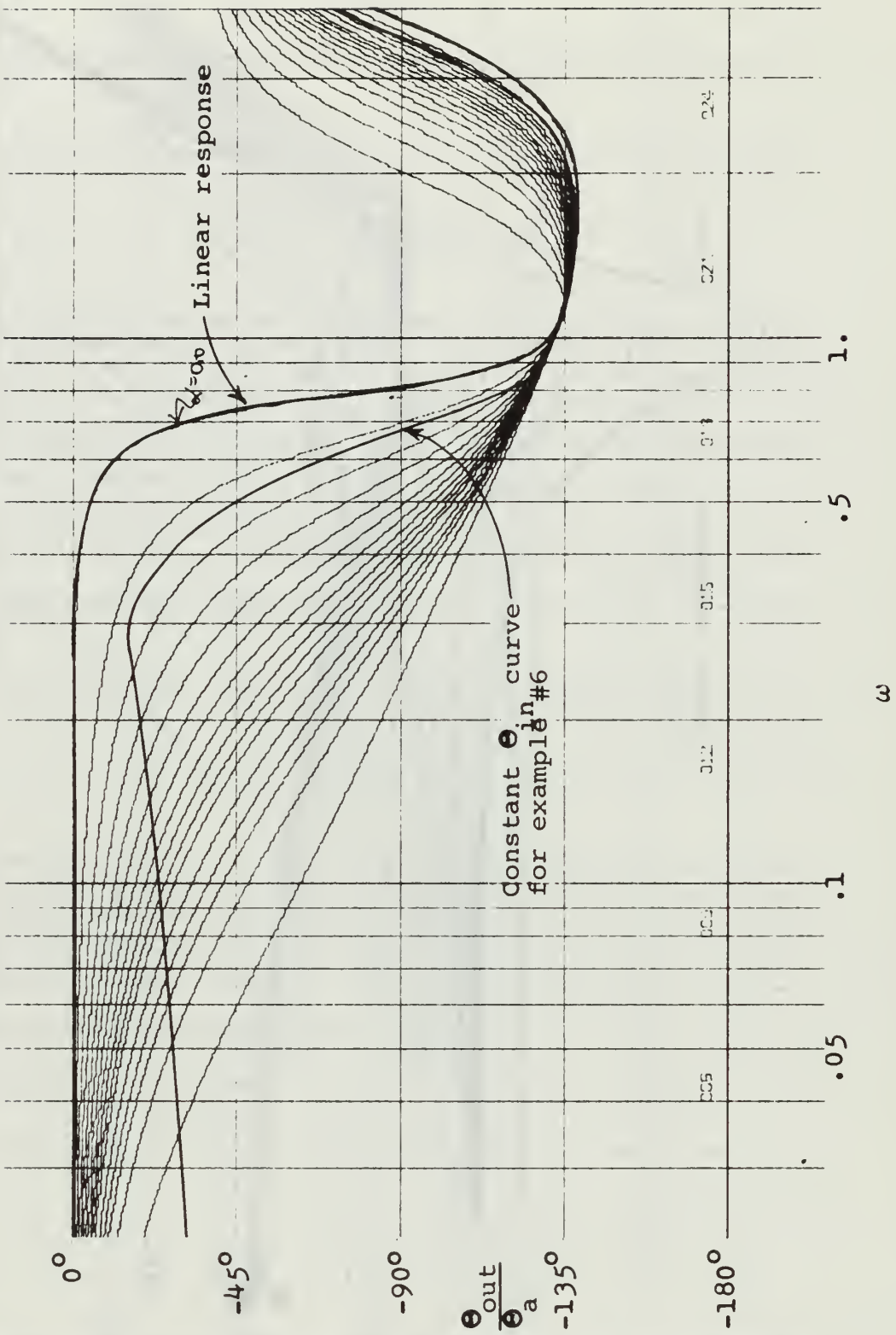
Figure 3.27 Closed loop phase frequency response of example #6

100

IV. THE EFFECT OF CHANGING THE INPUT

A.   INTRODUCTION

In the past examples, the frequency response was de-
scribed as a constant Θ_{in} curve. The frequency response
was obtained by letting Θ_{in} be equal to some specified
value. A change in Θ_{in} results in a change in the response.
In this chapter, two examples are analyzed for different
values of input. The first example has a saturation ele-
ment in the forward path, and the second example has a
dead zone element in the major feedback path.

B.   THE EFFECT OF CHANGING THE INPUT OF A SYSTEM WITH

A SATURATION ELEMENT

The example of chapter 2 used an input of magnitude
10.0, Θ_{in} = 10.0. To see what is the effect of various
other amplitudes of signals, Θ_{in} of 5.0, 7.5, 10.0, 15.0,
20.0 and 25.0 will be used. The steps in this analysis
are the same as those taken in previous examples. In order
to derive a relationship between gain of the saturation
element and frequency, the transfer function of the signal
going into the saturation element must be obtained. By
simple manipulation $\frac{\Theta_e}{\Theta_{in}}$ can be found, as shown in the fol-
lowing steps:

$$\Theta_e = \Theta_{in} - \Theta_{out} \qquad\qquad (4.1)$$
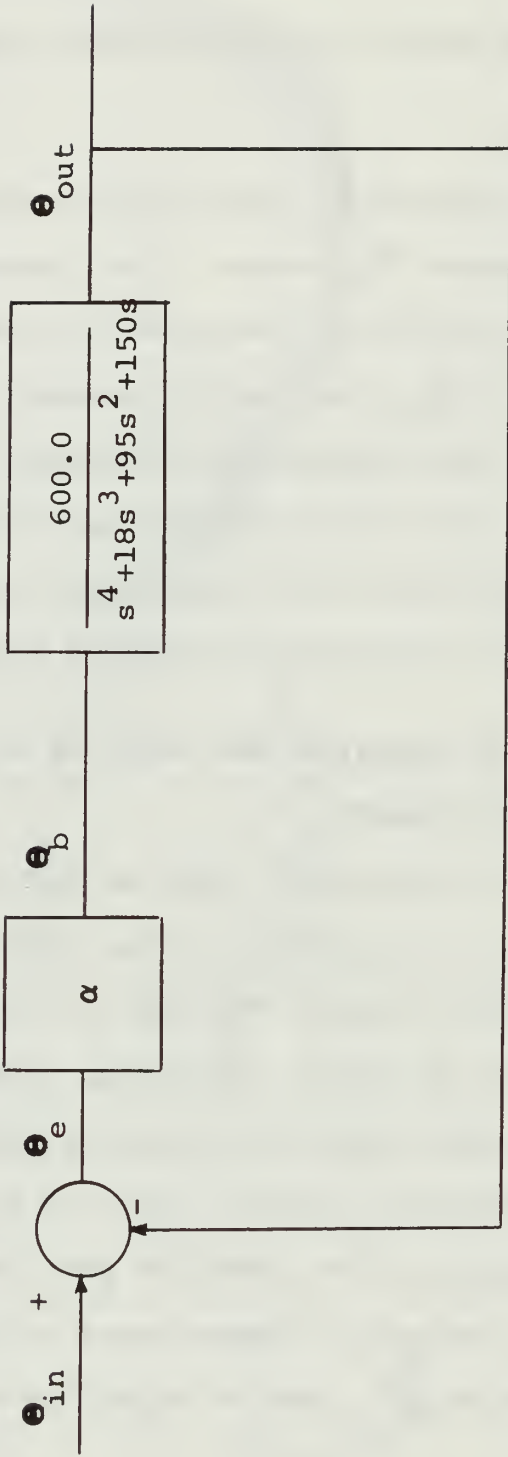
Figure 4.1 Block diagram of servomechanism in chapter 2

Dividing by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 1 - \frac{\Theta_{out}}{\Theta_{in}}$$

$$= 1 - \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \qquad (4.2b)$$

$$= \frac{s^4 + 18s^3 + 95s^2 + 150s}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \qquad (4.2c)$$

where α is the variable gain of the saturation element.

By using the PARAM program mentioned previously, a plot of α, (variable gain of the saturation element), versus the magnitude of $\frac{\Theta_e}{\Theta_{in}}$ is obtained. This is shown in figure (4.2). The saturation element may be represented by a variable gain which is a function of the input signal, Θ_e. However the plot in figure (4.2) is gain, α, versus $\frac{\Theta_e}{\Theta_{in}}$. By the following equations, a relationship between α and $\frac{\Theta_e}{\Theta_{in}}$ is developed.

$$\alpha = \frac{\Theta_b}{\Theta_e} \qquad (4.3)$$

where Θ_b is the signal out of the saturation element as shown in figure (4.3).

Dividing both the numerator and the denominator of the right-hand side of equation (4.3) by Θ_{in},

$$\alpha = \frac{\Theta_b / \Theta_{in}}{\Theta_e / \Theta_{in}} \qquad (4.4)$$

103
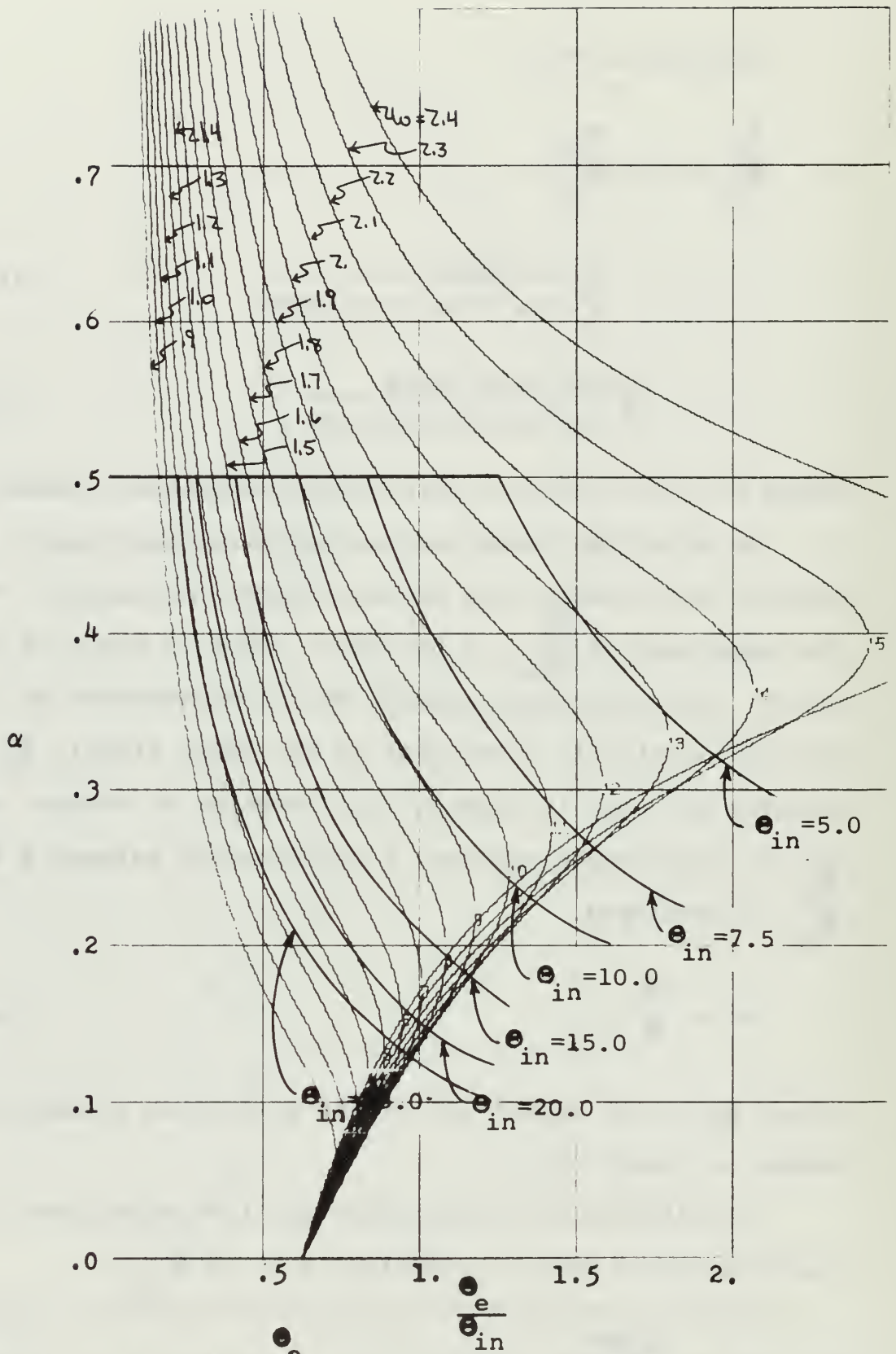
Figure 4.2 α versus $\dfrac{\theta_e}{\theta_{in}}$ with constant ω curves and constant θ_{in} curves
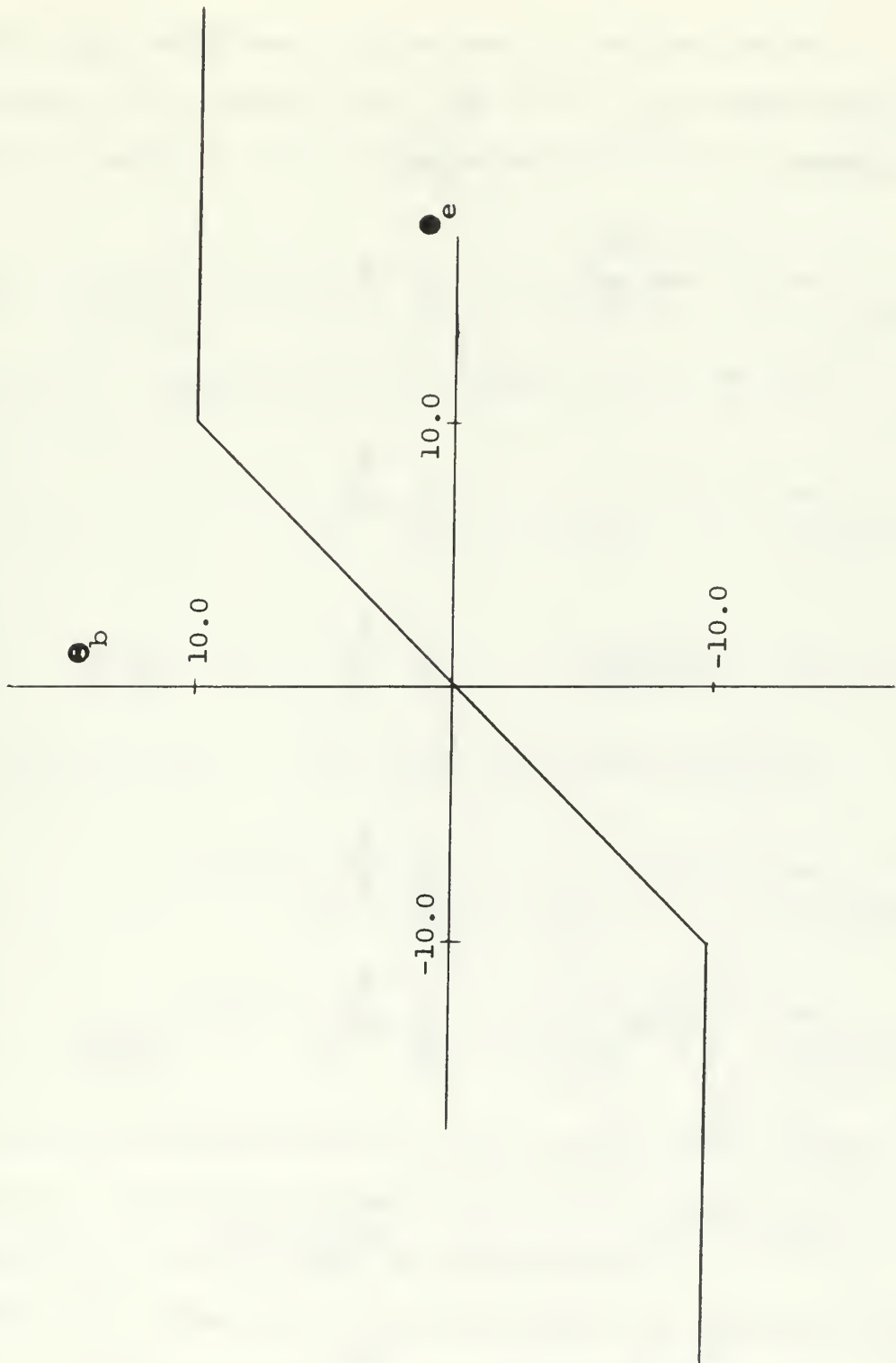
Figure 4.3 Saturation element in servomechanism

α is constant in the region where $\Theta_e \leq 10.0$. In the region where $\Theta_e \geq 10.0$, Θ_b, the output of the saturation element, is 10.0. Thus equation (4.4) becomes,

$$\alpha = \frac{10.0/\Theta_{in}}{\Theta_e/\Theta_{in}} \qquad \Theta_e \geq 10.0 \qquad (4.5)$$

For $\Theta_{in} = 5.0$, equation (4.4) becomes

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 2.0 \qquad (4.62)$$

$$\alpha = \frac{2.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 2.0 \qquad (4.6b)$$

In like manner, for $\Theta_{in} = 7.5$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 1.333 \qquad (4.7a)$$

$$\alpha = \frac{1.333}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 1.333 \qquad (4.7b)$$

For $\Theta_{in} = 10$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 1.0 \qquad (4.8a)$$

$$\alpha = \frac{1.0}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 1.0 \qquad (4.8b)$$

For $\Theta_{in} = 20.0$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.50 \qquad (4.9a)$$

$$\alpha = \frac{0.5}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.50 \qquad (4.9b)$$

For $\Theta_{in} = 15.0$

$$\alpha \quad 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.667 \qquad (4.10a)$$

$$\alpha = \frac{0.667}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.667 \qquad (4.10b)$$

For $\Theta_{in} = 25.0$

$$\alpha = 1.0 \qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.40 \qquad (4.11a)$$

$$\alpha = \frac{0.40}{\Theta_e/\Theta_{in}} \qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.40 \qquad (4.11b)$$

By using equations (4.6), (4.7), (4.8), (4.9), (4.10) and (4.11), a table may be made up of β as a function of $\frac{\Theta_e}{\Theta_{in}}$ and Θ_{in}. This is shown in table (IV.1).

Using the values of α in table (IV.1), the constant Θ_{in} curves are drawn on figure (4.2), using α and $\frac{\Theta_e}{\Theta_{in}}$ as coordinates.

		α				
$\dfrac{\Theta_e}{\Theta_{in}}$	$\Theta_{in}=5.0$	$\Theta_{in}=7.5$	$\Theta_{in}=10.0$	$\Theta_{in}=15.0$	$\Theta_{in}=20.0$	$\Theta_{in}=25.0$
0.0	1.0	1.0	1.0	1.0	1.0	1.0
0.1	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.0	1.0	1.0	1.0	1.0	1.0
0.3	1.0	1.0	1.0	1.0	1.0	1.0
0.4	1.0	1.0	1.0	1.0	1.0	1.0
0.5	1.0	1.0	1.0	1.0	1.0	0.8
0.6	1.0	1.0	1.0	1.0	0.835	0.667
0.7	1.0	1.0	1.0	0.955	0.715	0.572
0.8	1.0	1.0	1.0	0.835	0.625	0.50
0.9	1.0	1.0	1.0	0.742	0.556	0.445
1.0	1.0	1.0	1.0	0.667	0.5	0.4
1.0	1.0	1.0	0.91	0.606	0.455	0.364
1.2	1.0	1.0	0.833	0.556	0.417	0.334
1.3	1.0	1.0	0.77	0.514	0.385	0.308
1.4	1.0	0.955	0.714	0.477	0.357	0.286
1.5	1.0	0.89	0.667	0.445	0.333	0.267
1.6	1.0	0.835	0.625	0.417	0.312	0.25
1.7	1.0	0.785	0.588	0.393	0.294	0.235
1.8	1.0	0.741	0.555	0.371	0.278	0.222
1.9	1.0	0.702	0.526	0.352	0.263	0.212
2.0	1.0	0.669	0.500	0.334	0.250	0.200
2.1	0.95	0.633	0.476			
2.2	0.91	0.606	0.455			
2.3	0.87	0.55	0.435			
2.4	0.83	0.536	0.433			
2.5	0.80	0.534	0.417			
2.6	0.77	0.514	0.400			
2.7	0.74	0.494				
2.8	0.715	0.476				
2.9	0.69	0.460				
3.0	0.667	0.445				

Table IV.1 α as a Function of $\dfrac{\Theta_e}{\Theta_{in}}$ and Θ_{in} for the

Saturation Element

The intersection of the constant Θ_{in} curves and ω curves gives the relationship between α, the variable gain of the saturation element, and ω, frequency. It should be noted that each constant Θ_{in} curve intersects a particular constant ω curve two or three times, giving two or three values of α for the same frequency. This is the jump resonance effect which will be shown more clearly in the closed loop frequency response curves.

Using the PARAMS program again, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ versus ω is obtained. This plot, figure (4.4), consists of constant α curves of various values between $\alpha = 0.25$ and $\alpha = 1.0$. The coordinates of the constant Θ_{in} curves in figure (4.2) are ω, frequency, and α, nonlinear gain. Using these coordinates, the constant Θ_{in} curves are replotted in figure (4.4). As was noted previously in figure (4.2), each constant Θ_{in} curve intersects some ω curves more than once. When plotted on figure (4.4), this effect is clearly seen as being a jump resonance effect caused by the saturation element.

The effect of changing the input is seen in figure (4.4). Increasing Θ_{in} decreases the magnitude of the resonance peak and decreases the resonance frequency. The jump resonance remains about the same as Θ_{in} changes.
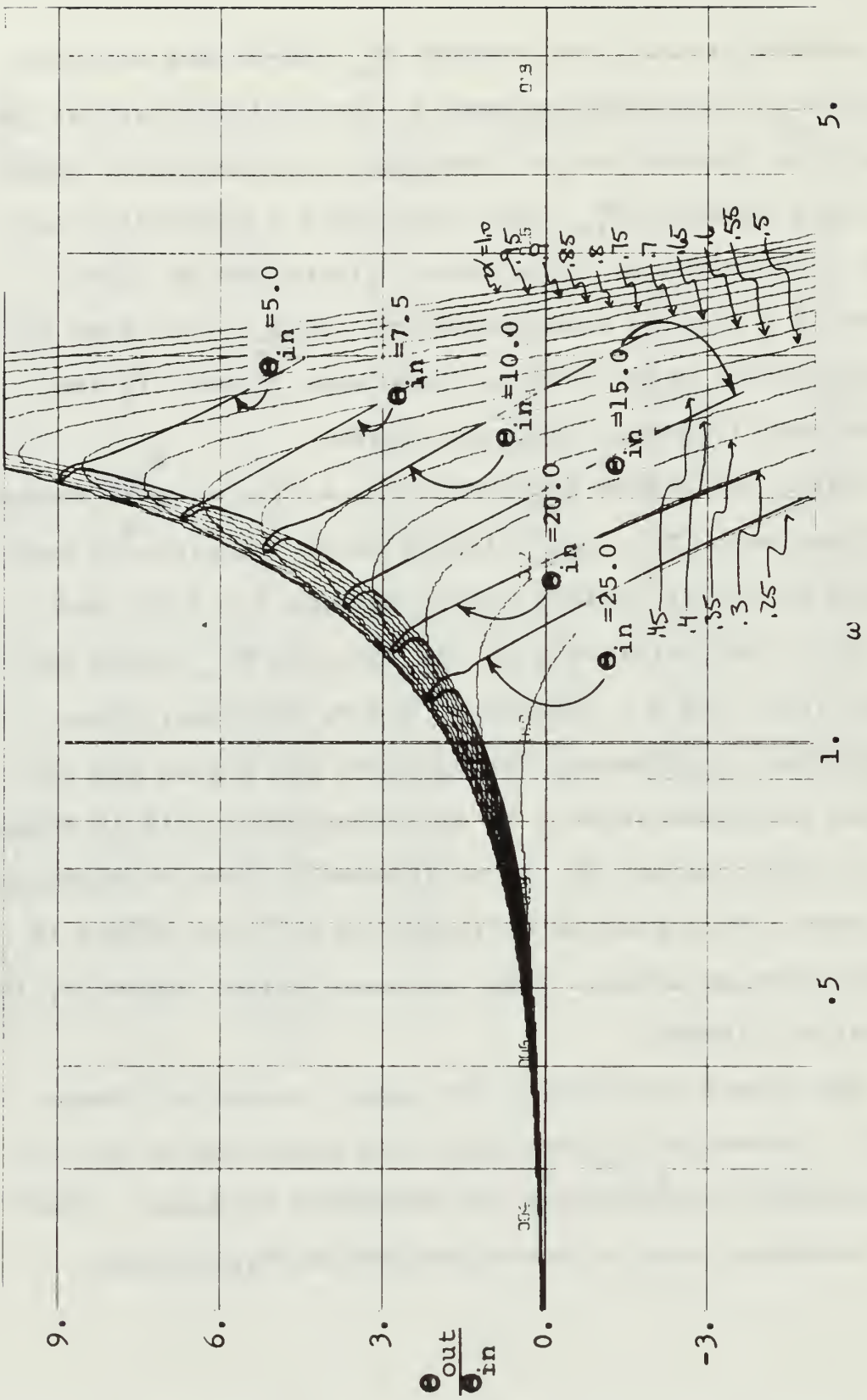
Figure 4.4 Closed loop frequency response for various Θ_{in}

C.   THE EFFECT OF CHANGING THE INPUT OF A SYSTEM WITH

     A DEAD ZONE ELEMENT

The effect of changing the input of the system with a dead zone element can be seen in the same manner as the previous example.   The block diagram of the system with dead zone is figure (4.5), where β is the variable gain of the dead zone element.   The dead zone element is different from the one used in chapter 2.   Its input-output describing curve is shown in figure (4.6) where Θ_{out} is the signal out of the element.   Note that the non-dead zone portion of the curve does not have unity gain.   The gain is 2.0 when the signal is not in the dead zone portion.

The frequency of the system will be analyzed for four different inputs.   These inputs are Θ_{in} = 10.0. Θ_{in} = 20.0, Θ_{in} = 40.0   and Θ_{in} = 80.0.   In order to analyze the problem, the same steps must be taken as in the preceding examples.   The first step is to find a relationship between $\dfrac{\Theta_{out}}{\Theta_{in}}$ and β, the nonlinear variable gain of the saturation element.

Referring to figure (4.5), it is seen that,

$$\beta = \frac{\Theta_a}{\Theta_{out}}$$

However β = 0.0 when $\Theta_{out} \leq 40.0$ from figure (4.6). Or,

$$\Theta_a = 2(\Theta_{out}-40) = 2\Theta_{out}-80. \qquad \Theta_{out} \geq 40.0 \qquad (4.13a)$$
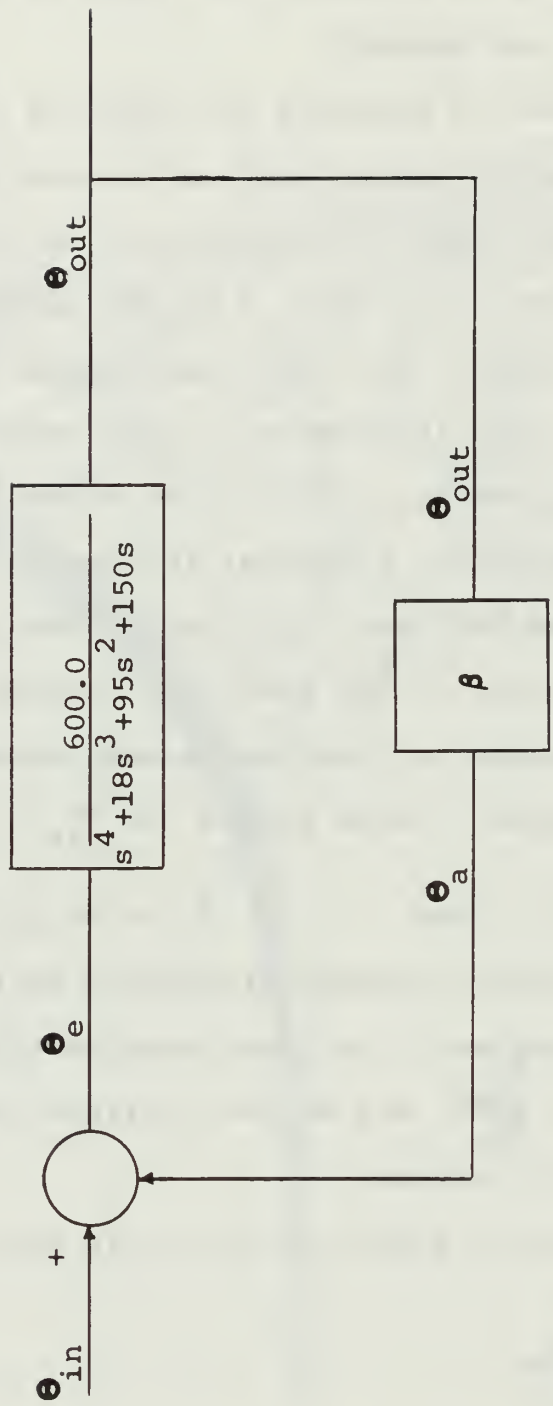
Figure 4.5 Block diagram of servomechanism with dead
zone element in the feedback path
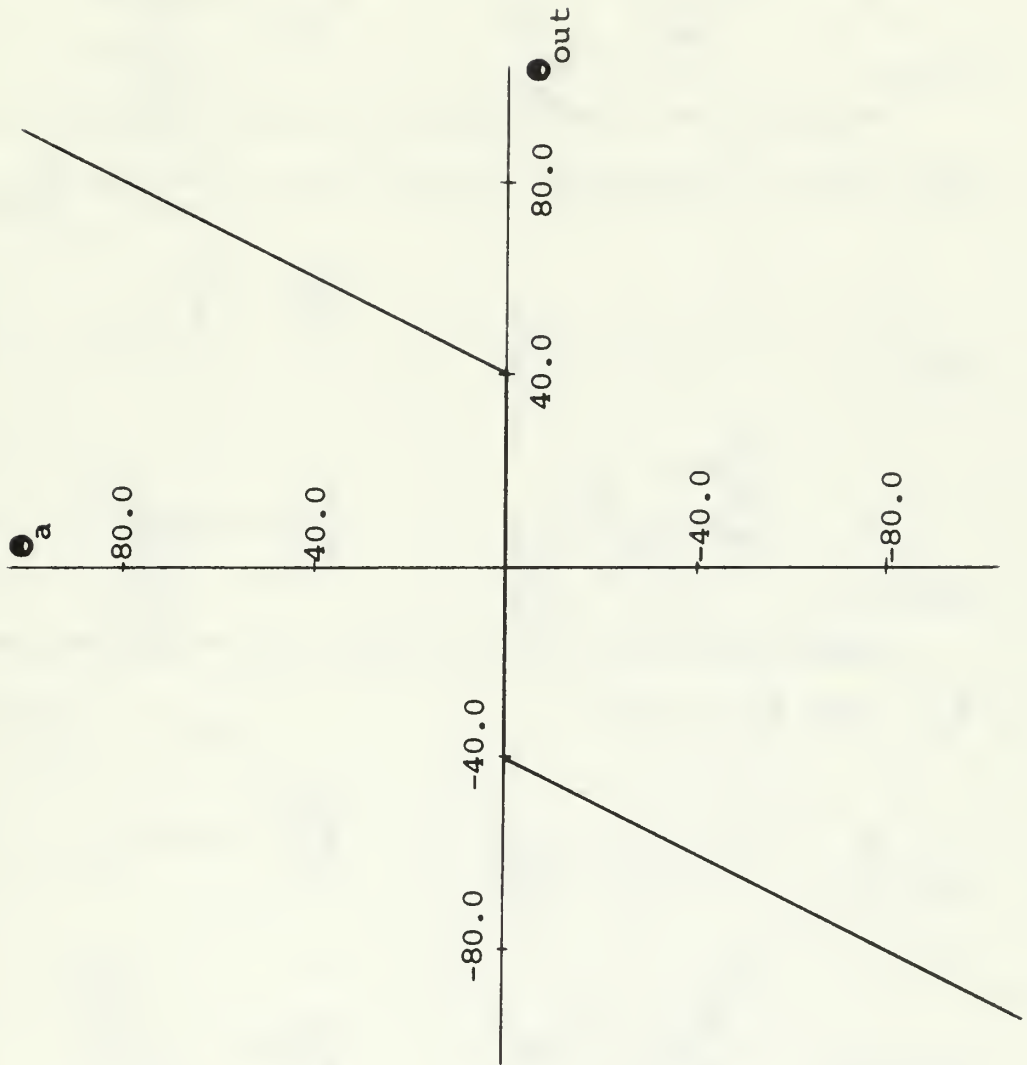
Figure 4.6 Dead zone element of second example

113

Substituting equation (4.14) into equation (4.12),

$$\beta = \frac{2\Theta_{out} - 80}{\Theta_{out}} \qquad\qquad \Theta_{out} \geq 40. \qquad\qquad (4.13b)$$

Dividing both numerator and denominator of the right-hand sides of equations (4.13a) and (4.13b) by Θ_{in},

$$\beta = 0.0 \qquad\qquad \frac{\Theta_{out}}{\Theta_{in}} \leq \frac{40}{\Theta_{in}} \qquad\qquad (4.15a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - \dfrac{80}{\Theta_{in}}}{\Theta_{out}/\Theta_{in}} \qquad\qquad \frac{\Theta_{out}}{\Theta_{in}} \geq \frac{40.0}{\Theta_{in}} \qquad\qquad (4.15b)$$

Equations (4.15a) and (4.15b) can be solved for each Θ_{in}. For $\Theta_{in} = 10.0$,

$$\beta = 0.0 \qquad\qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 4.0 \qquad\qquad (4.16a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 8.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad\qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 4.0 \qquad\qquad (4.16b)$$

For $\Theta_{in} = 20.0$,

$$\beta = 0.0 \qquad\qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 2.0 \qquad\qquad (4.17a)$$

114

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 4.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 2.0 \qquad (4.17b)$$

For $\Theta_{in} = 40.0$,

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 1.0 \qquad (4.18a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 2.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 1.0 \qquad (4.18b)$$

For $\Theta_{in} = 80.0$,

$$\beta = 0.0 \qquad \frac{\Theta_{out}}{\Theta_{in}} \leq 0.5 \qquad (4.19a)$$

$$\beta = \frac{\dfrac{2\Theta_{out}}{\Theta_{in}} - 1.0}{\dfrac{\Theta_{out}}{\Theta_{in}}} \qquad \frac{\Theta_{out}}{\Theta_{in}} \geq 0.5 \qquad (4.19b)$$

Equations (4.16a), (4.16b), (4.17a), (4.17b), (4.18a), (4.18b), (4.19a) and (4.19b) give the relation between $\frac{\Theta_{out}}{\Theta_{in}}$ and β for the various constant Θ_{in}. This relation is made into table (IV.2) where $\frac{\Theta_{out}}{\Theta_{in}}$ is the entering value and β is read out for a constant Θ_{in}.

115

$$\beta$$

$\dfrac{\Theta_{out}}{\Theta_{in}}$	$\Theta_{in} = 10.$	$\Theta_{in} = 20.$	$\Theta_{in} = 40.$	$\Theta_{in} = 80.$
0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	1.0
1.5	0.0	0.0	0.667	1.33
2.0	0.0	0.0	1.00	1.5
2.5	0.0	0.4	1.20	1.6
3.0	0.0	0.667	1.33	
3.5	0.0	0.855	1.43	
4.0	0.0	1.00	1.5	
4.5	0.22	1.12	1.6	
5.0	0.4	1.2		
5.5	0.546	1.27		
6.0	0.667	1.33		
6.5	0.77	1.38		
7.0	0.856	1.43		
7.5	0.93	1.47		
8.0	1.00	1.50		
8.5	1.06	1.53		
9.0	1.11	1.56		
9.5	1.16	1.58		
10.0	1.20	1.60		
10.5	1.24			
11.0	1.27			
11.5	1.30			
12.0	1.33			

Table IV.2 β as a Function of $\dfrac{\Theta_{out}}{\Theta_{in}}$ and Θ_{in} for the Dead Zone

Using the PARAMS-5 subprogram of the PARAMS program, a plot of β versus $\frac{\Theta_{out}}{\Theta_{in}}$ with constant ω curves is obtained, as shown in figure (4.7). Using the relation between $\frac{\Theta_{out}}{\Theta_{in}}$ and β, tabulated in table (IV.2), constant Θ_{in} curves are drawn on figure (4.7). The intersection of the constant Θ_{in} curves and the ω curves in figure (4.7) gives the relation between frequency and β.

The last step in the analysis, is the drawing of the closed loop frequency response for the various inputs. Using the PARAMS-7 subprogram of the PARAMS program, a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus frequency with constant β curves is obtained. This is figure (4.8). Using the relation between β and ω just found, the constant Θ_{in} curves are drawn on figure (4.8).

Figure (4.8) shows the effect of the various inputs. As Θ_{in} is decreased, the magnitude of the flat part of the response is decreased. At the same time, a jump resonance effect takes place. This jump resonance is opposite to the one found for the saturation problem of this chapter.
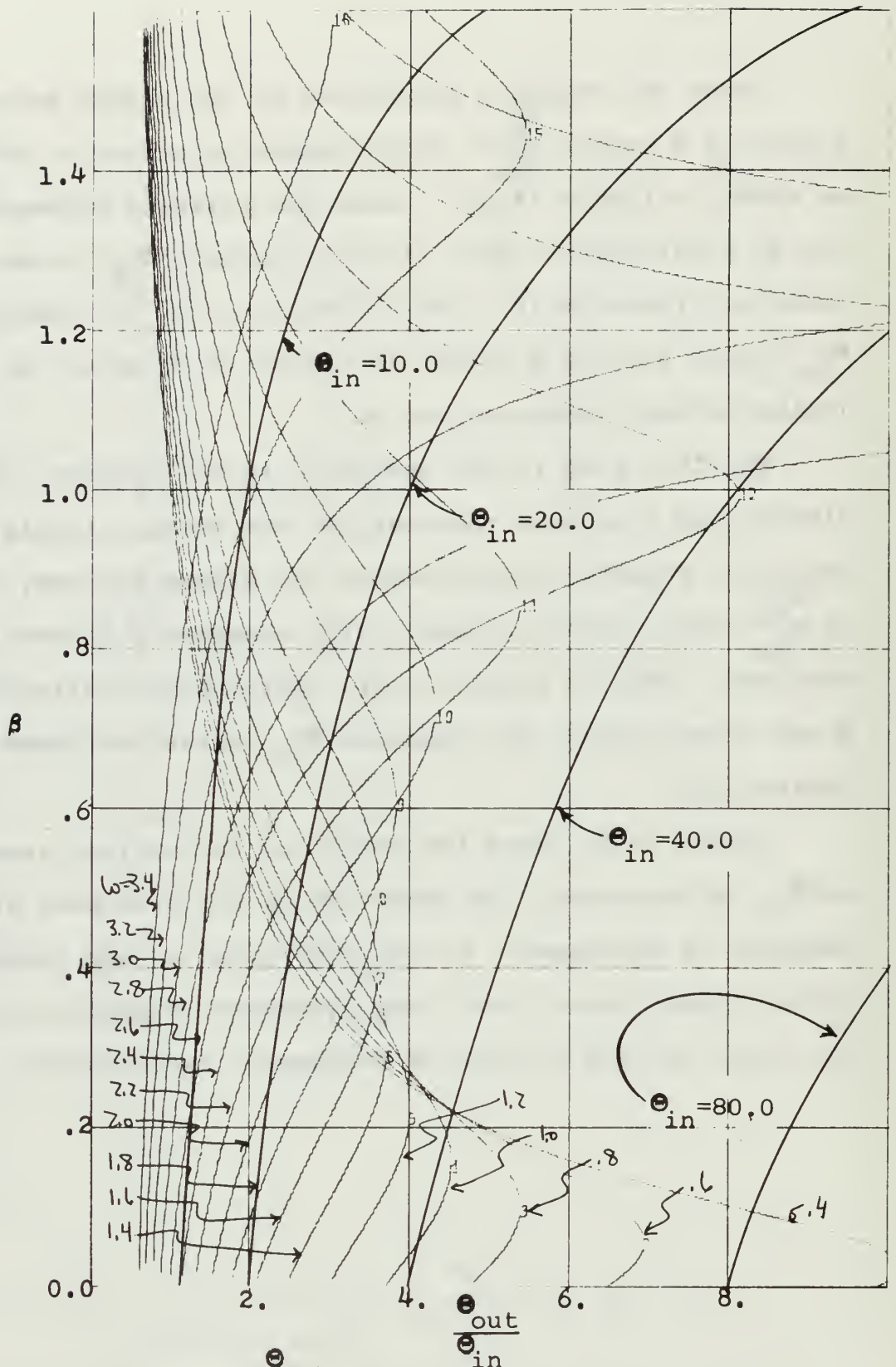
Figure 4.7 β versus $\dfrac{\Theta_{out}}{\Theta_{in}}$ magnitude for second example

118
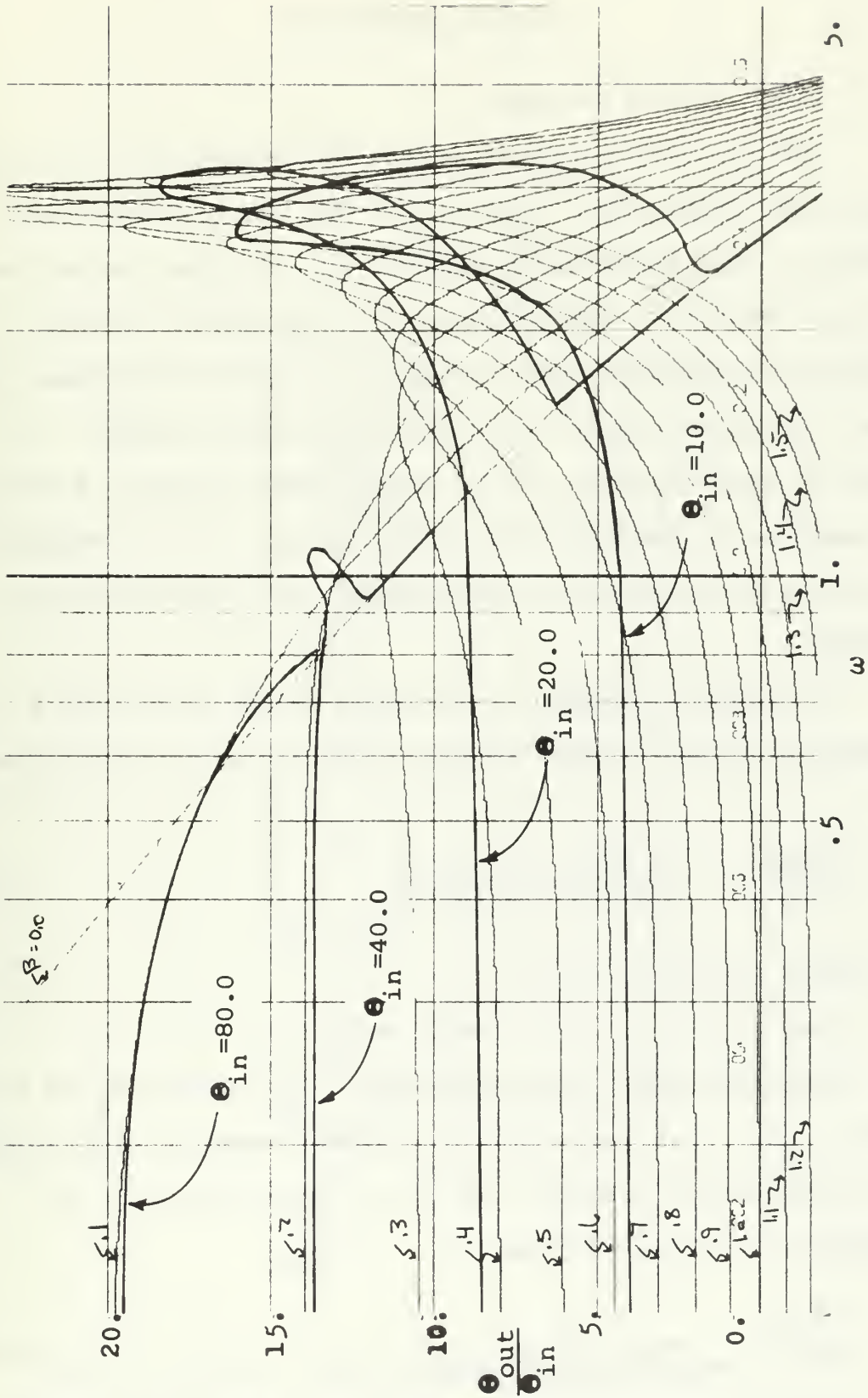
Figure 4.8 Closed loop frequency response for various θ_{in} in a system with dead zone

V. DESIGN TECHNIQUES

A. FIRST DESIGN PROBLEM

The previous chapters explored the techniques of ana-
lysis and showed that they worked for every example at-
tempted.   The question is whether or not these techniques
can be reversed.   The problem is to start with a given
linear system, and design an element, linear or nonlinear, so
that the system meets a certain specified response.   It
must be kept in mind that if a nonlinear element is used,
it must be a signle-valued nonlinearity.   To see whether or
not this design can be accomplished, an example will be
worked.

The design problem is a simple third order type 1
feedback system, where the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{K_1 K_2}{s(s+p_1)(s+p_2)+K_1 K_2} \qquad (5.1)$$

as shown in figure (5.1).

Let $p_1 = 1$, $p_2 = 2$, and $K_1 = 1$.

The first part of the problem is to find a K_2 so that
the system is at the stability limit.   This is easily done
by conventional methods, and it is found that $K_2 = 6$.
Equation (5.1) now becomes,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{6}{s^3 + 3s^2 + 2s + 6} \qquad (5.2)$$

120
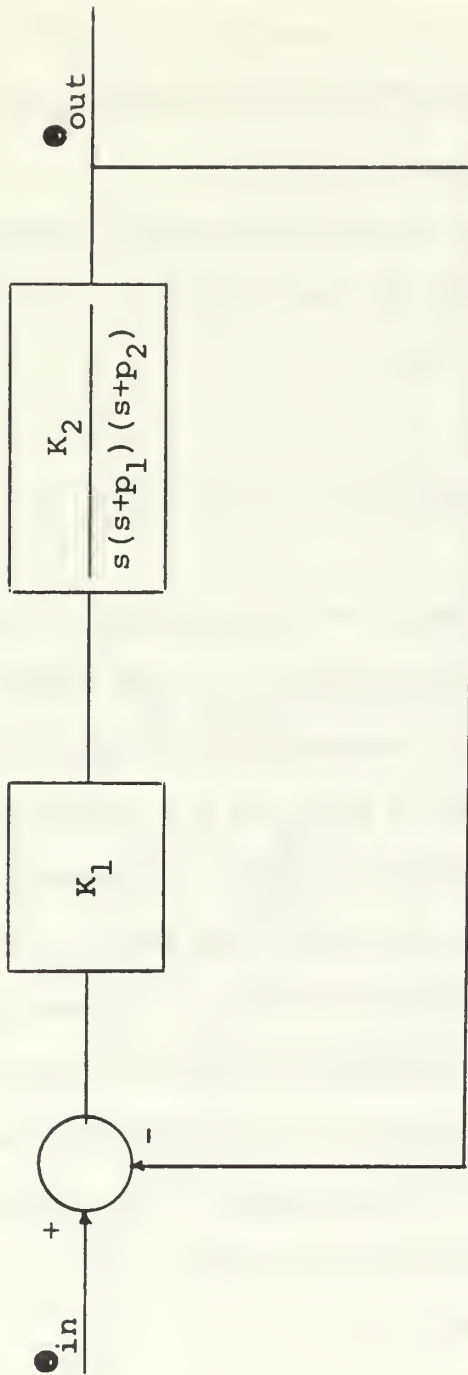
Figure 5.1 Type 1 third order feedback system

The second part of the design problem is also linear. The requirement is to provide tachometer feedback in a minor loop so that $M_p(\omega) = 2.0$. The block diagram of the system now is figure (5.2).

As has been previously shown, the PARAMS program can solve this aspect of the problem.[4] The transfer function of the system is now,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{6}{s^3 + 3s^2 + 2s + 6 + 6K_t s} \tag{5.3}$$

where K_t is the gain of the tachometer feedback element.

The PARAM-5 subprogram of the PARAMS program provides an easy method for determining K_t when $M_p(\omega) = 2.0$. Figure (5.3) gives a plot of K_t, tachometer feedback gain, versus the magnitude of $\frac{\Theta_{out}}{\Theta_{in}}$. Constant ω curves have been drawn on the plot for the region of frequency that is of interest. A vertical line is drawn at $\frac{\Theta_{out}}{\Theta_{in}} = 2.0$. This line represents the desired $M_p(\omega)$. The intersection of any horizontal line with the constant ω curves gives the frequency response of the system. The ordinate value of this line is K_t. Thus the horizontal line at $K_t = 0.35$ gives the desired $M_p(\omega) = 2.0$.

---

[4] Glavis, G.O., _Frequency Response in the Parameter Plane_, Master's Thesis, Naval Postgraduate School, 1967.
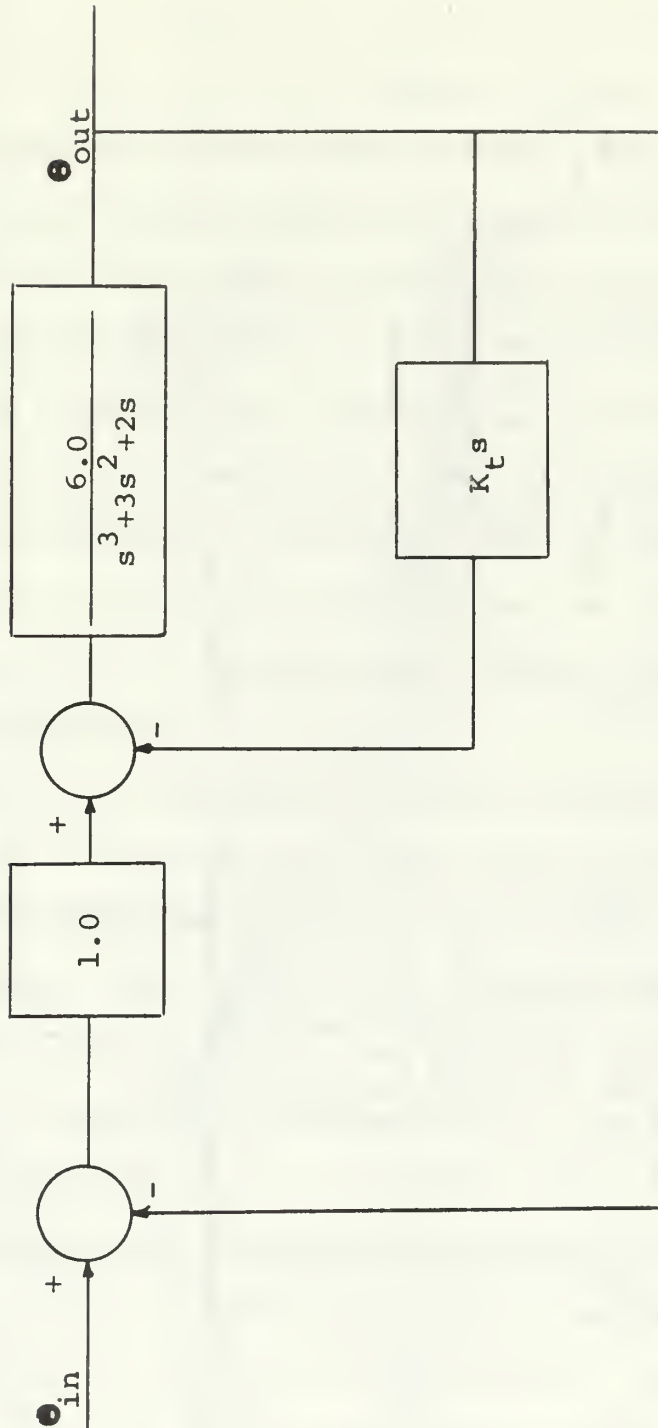
Figure 5.2 Third order feedback system with tackometer feedback
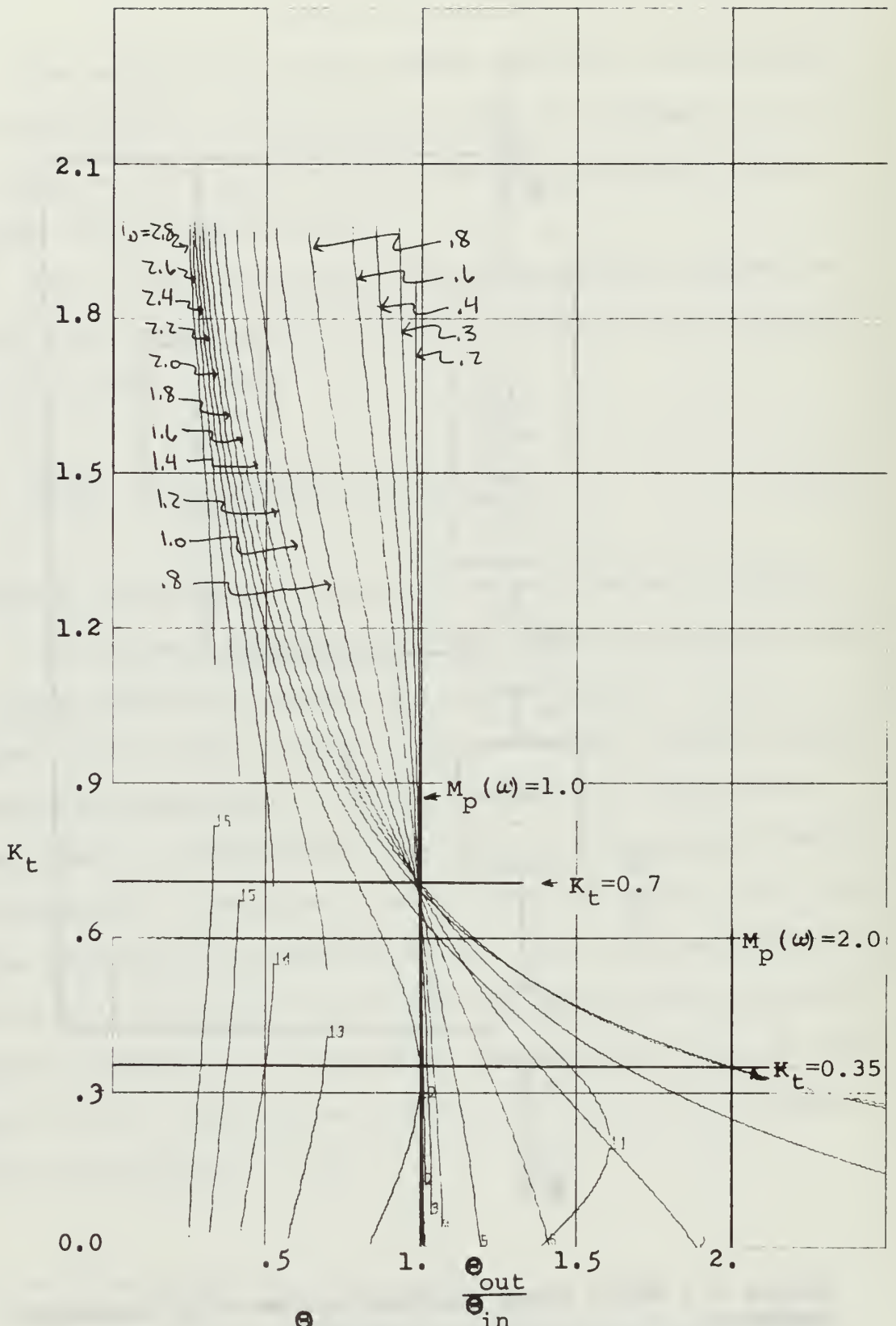
Figure 5.3 K_t versus $\dfrac{\Theta_{out}}{\Theta_{in}}$ magnitude for design problem

124

The last step in the design is to change $M_p(\omega)$ so that

$M_p(\omega) = 1.2$. This is to be done by putting either a non-

linear or linear element in the minor feedback path. This

additional requirement changes the block diagram so that it

now looks like figure (5.4), where β represents the gain

of the nonlinear or linear element. Many techniquess can be

used for designing a compensator to accomplish this new re-

quirement. However in this particular case, a restriction

will be placed on the system. The requirement is that the

compensator be either a constant gain device, i.e., an am-

plifier or attenuator, or a variable gain device, such as a

single-valued nonlinearity.

The next step in this design is similar to the pre-

vious step. A plot of nonlinear or linear gain (or attenu-

ation) versus $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude ($M_p(\omega)$) is to be made. This

is done by the PARAM-5 subprogram of the PARAMS program.

The plot is shown as figure (5.5). The ordinate is β, the

linear or nonlinear gain, which ranges from 0.0 to 2.1.

The abscissa is $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude. A vertical line is drawn

at $\frac{\Theta_{out}}{\Theta_{in}} = 1.2$. Because the requirements of the problem

state that $M_p(\omega) = 1.2$, no design solution can extend to

the right of the $\frac{\Theta_{out}}{\Theta_{in}} = 1.2$ line in figure (5.5). This

region is shaded, and is a 'restricted design zone' as far

as the design is concerned.

The final result in this design is to develop the shape

of an element which, when put in the system for β in figure
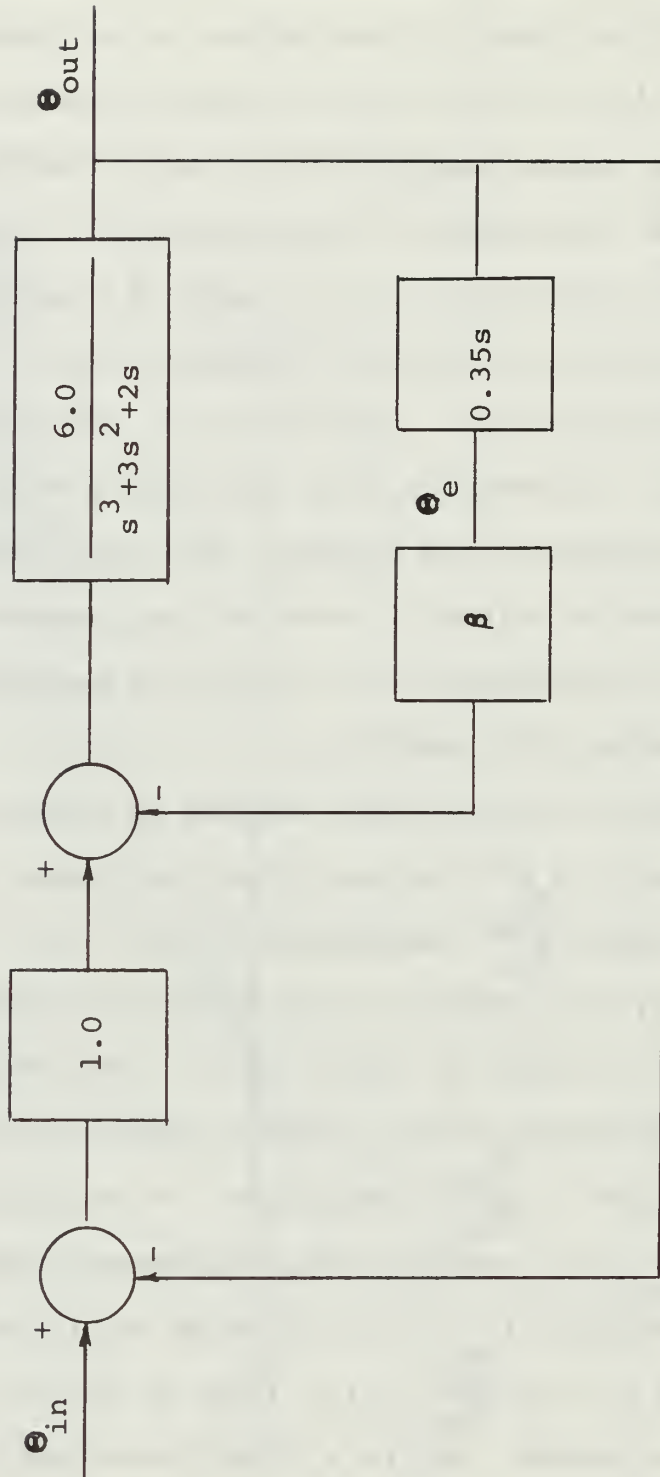
Figure 5.4 Third order system with linear/nonlinear
compensator in tackometer feedback loop

Figure 5.5 β versus $\dfrac{\Theta_{out}}{\Theta_{in}}$ magnitude for design problem

127
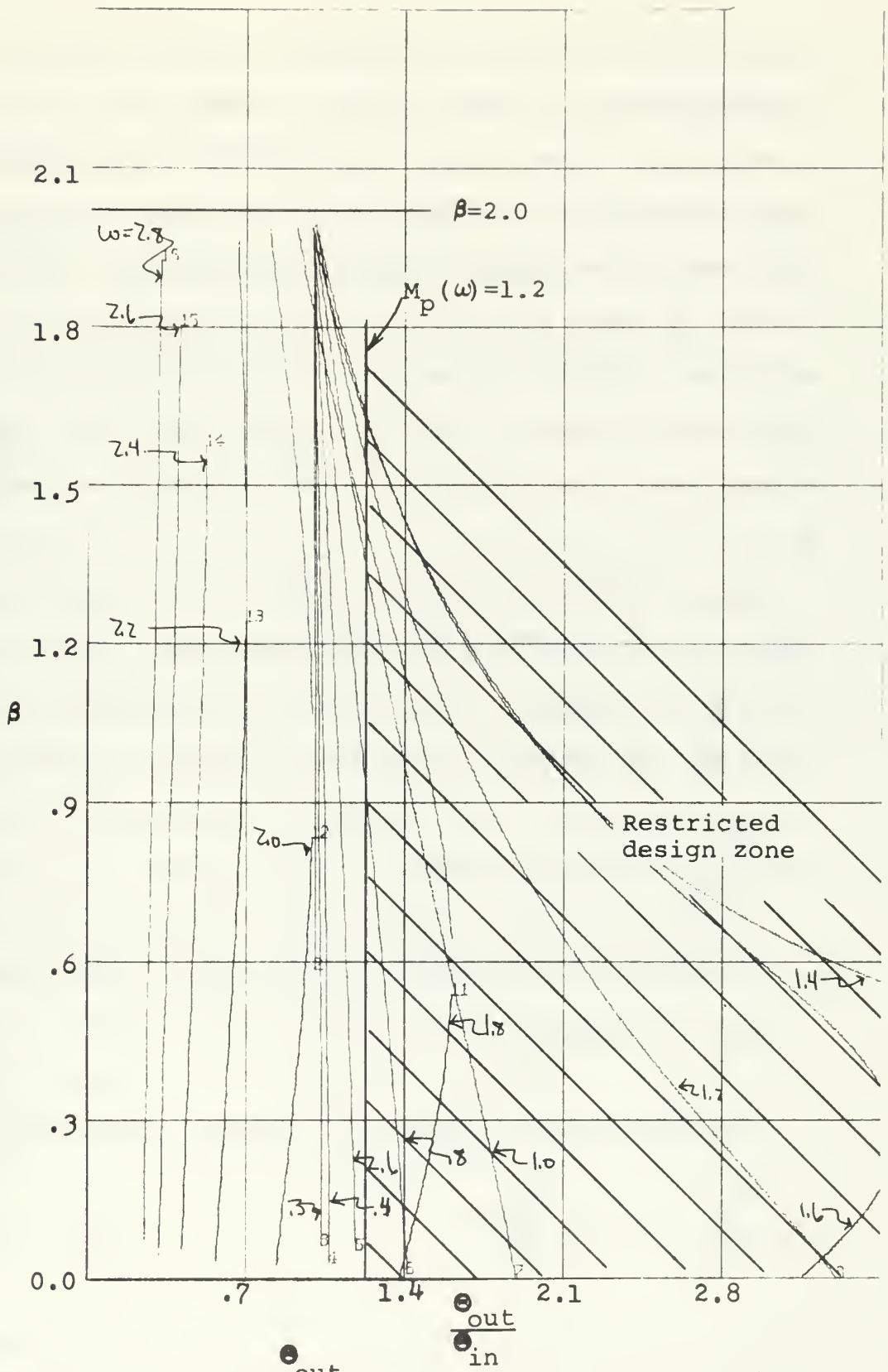
(5.4), gives the desired response. Before this can be

obtained, steps similar to those in chapters 2,3 and 4 must

be followed. These steps, however, must be reversed.

Having determined in figure (5.5) the region in which de-

sign cannot take place, a relationship between this region

and the variable β is found. The relationship has the co-

ordinates of β and frequency. The next step is to find a

relationship between β and the signal going into the linear

or nonlinear element, using the relationship between β and

ω.

It is difficult to find a relationship between a vari-

able gain, β, and Θ_e in terms of frequency. However, it is

a relatively simple process to find a relationship between

β and $\dfrac{\Theta_e}{\Theta_{in}}$ in terms of frequency. Looking at figure (5.4),

$$\frac{\Theta_e}{\Theta_{out}} = K_t s = 0.35s \qquad (5.4)$$

Multiplying both sides of equation (5.4) by Θ_{out},

$$\Theta_e = .35s \; \Theta_{out} \qquad (5.5)$$

And dividing both sides of equation (5.5) by Θ_{in},

$$\frac{\Theta_e}{\Theta_{in}} = 0.35s \; \frac{\Theta_{out}}{\Theta_{in}} \qquad (5.6)$$

128

By substituting $K_t = 0.35$ into equation (5.3) and letting the nonlinear or linear element be represented by β, equation (5.3) becomes,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{6}{s^3 + 3s^2 + 2s + 6 + 2.1\beta s} \qquad (5.7)$$

Substituting equation (5.7) into equation (5.6),

$$\frac{\Theta_e}{\Theta_{in}} = \frac{2.1s}{s^3 + 3s^2 + 2s + 6 + 2.1\beta s} \qquad (5.8)$$

Equation (5.8) is used in the PARAM-5 subprogram to produce figure (5.6). Figure (5.6) has β, the variable gain, as the ordinate and $\frac{\Theta_e}{\Theta_{in}}$ magnitude as the abscissa. The same constant ω curves that are on figure (5.5) are on figure (5.6). Using β and ω as the coordinates for the restricted design zone in figure (5.5), this zone can be transferred to figure (5.6). This restricted design zone represents the area in which design cannot take place. The line that is the left-hand boundary of the zone is the critical boundary which will be used to formulate a non-linear element.

Referring to figure (5.4) again, it is seen that,

$$\beta = \frac{\Theta_a}{\Theta_e} \qquad (5.9)$$

And that,

$$\Theta_a = \beta \Theta_e \qquad (5.10)$$
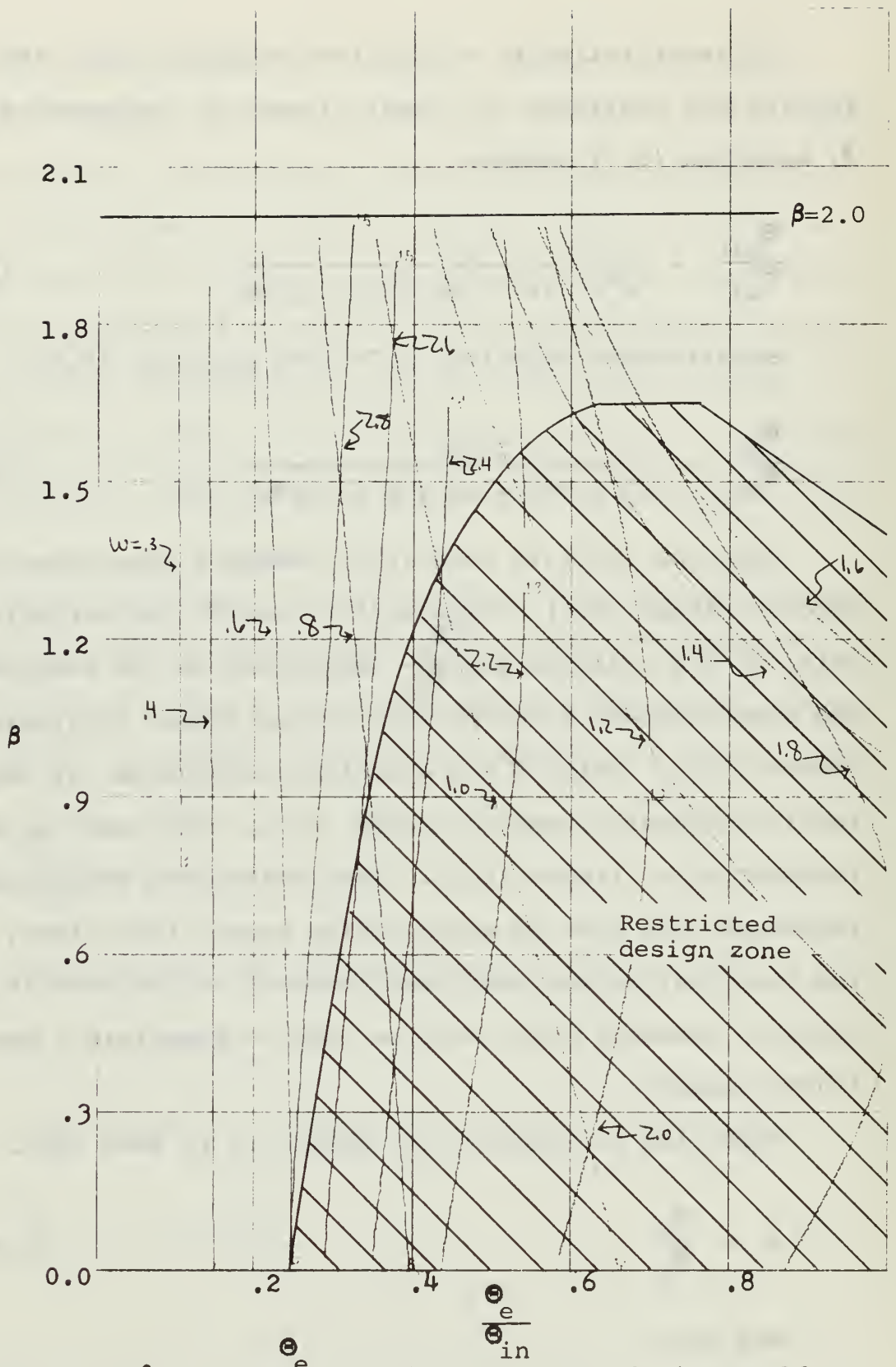
Figure 5.6 β versus $\frac{\Theta_e}{\Theta_{in}}$ magnitude for the design problem

Recalling that Θ_{in} is assumed a constant, the abscissa
of figure (5.6) is Θ_e when $\Theta_{in} = 1.0$. From equation (5.5)
and figure (5.6), the restricted design zone with its left-
hand boundary line can be reconstructed in a plot of Θ_a
versus Θ_e, where Θ_a is the signal out of the design ele-
ment.   This is depicted in figure (5.7) and provides a
ready means of design.

When looking at figure (5.7), a simple solution to the
problem of making $M_p(\omega) \leq 1.2$ is seen immediately. The so-
lution is to provide a constant gain of 2.0, or $\beta = 2.0$.
This constant gain is shown as a straight line in figure
(5.7). This solution could have been seen in figure (5.6)
by having a horizontal line at $\beta = 2.0$. The $\beta = 2.0$ line
in figure (5.6) does not pass through the restricted zone.
This solution is also seen in figure (5.5) as a $\beta = 2.0$
line also. One could go back in the design process to
where K_t was being determined, from figure (5.3), it is
seen that by making $K_t = .7$, a $M_p(\omega) = 1.0$ is obtained.
By making $\beta = 2.0$, one is making a new K_t, i.e.,
$K_t$new $= \beta K_t$ for a constant β.

However, looking at figure (5.7) again, it is seen
that a nonlinear design is also possible. One such non-
linear design is to have a relay with dead zone. This re-
lay is drawn on figure (5.7). For signals less than 0.25,
the output is 0.0. For signals greater than 0.25, the
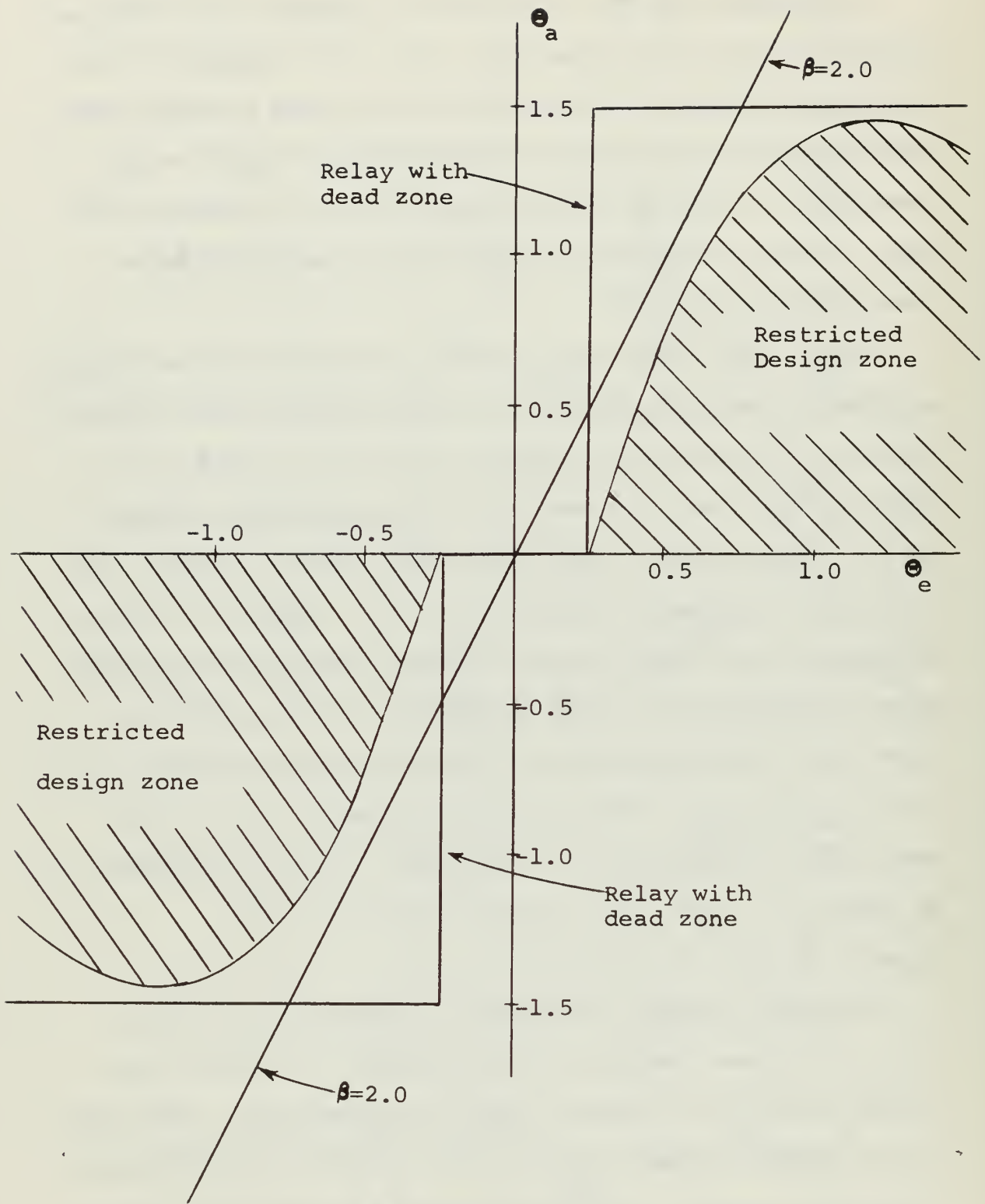
Figure 5.7 Plot of Θ_a-signal out of the element, versus Θ_e-signal into the element, with restricted design zone and proposed designs

132

output is 1.5. The relay with dead zone describing curve

(constant Θ_{in} curve) can be redrawn in the following

manner:

$$\beta = \frac{\Theta_a}{\Theta_e} \qquad (5.11)$$

which comes from figure (5.4). For this relay with dead

zone,

$$\Theta_a = 0.0 \qquad\qquad \Theta_e \leq 0.25 \qquad (5.12a)$$

$$\Theta_a = 1.5 \qquad\qquad \Theta_e \geq 0.25 \qquad (5.12b)$$

Substituting equation (5.12) into equation (5.11),

$$\beta = \frac{0.0}{\Theta_e} \qquad\qquad \Theta_e \leq 0.25 \qquad (5.13a)$$

$$\beta = \frac{1.5}{\Theta_e} \qquad\qquad \Theta_e \geq 0.25 \qquad (5.13b)$$

And dividing both the numerator and denominator of the

right-hand side of equation (5.13) by Θ_{in}, and letting

$\Theta_{in} = 1.0$ for part of the numerator,

$$\beta = 0.0 \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \leq 0.25 \qquad (5.14a)$$

$$\beta = \frac{1.5}{\Theta_e/\Theta_{in}} \qquad\qquad \frac{\Theta_e}{\Theta_{in}} \geq 0.25 \qquad (5.14b)$$

From equation (5.14), a table may be constructed simi-
lar to those in the previous chapters. The entering value
is $\frac{\Theta_e}{\Theta_{in}}$, and the value read off is β, the nonlinear vari-
able gain. $\frac{\Theta_e}{\Theta_{in}}$ is the abscissa of figure (5.6) and β is
the ordinate of figure (5.6). The result is table (V.1).

With table (V.1), the constant Θ_{in} curve for the relay
can be reconstructed on figure (5.6). However, since
figure (5.6) does not cover the range of β and $\frac{\Theta_e}{\Theta_{in}}$ needed,
a new plot is obtained. This plot is figure (5.8) where β
goes from 0.0 to 6.0 and $\frac{\Theta_e}{\Theta_{in}}$ goes from 0.0 to 2.4. It is
the same plot as figure (5.6), except that the scales have
been expanded. The restricted design zone is reconstructed
on figure (5.8). Now the constant Θ_{in} curve for the relay
with dead zone is plotted on figure (5.8). The intersec-
tion of the constant Θ_{in} curve and the ω curves gives the
relationship between β and ω. Using this relationship in
figure (5.8), the closed loop frequency response can be re-
constructed to show that the relay does meet the specifica-
tions.

The PARAM-7 subprogram of the PARAMS program is used
to reconstruct the closed loop frequency response. This
plot is figure (5.9) and has $\frac{\Theta_{out}}{\Theta_{in}}$ magnitude (dB) versus
ω , with constant β curves of β = 0.0, 0.5, 0.75, 1.0,
1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.5, 4.0, 5.0
and 6.0 drawn on it. The frequency response for the relay
is drawn on figure (5.9), using the coordinates of β and ω

$\dfrac{\Theta_e}{\Theta_{in}}$	β
0.0	0.0
0.1	0.0
0.2	0.0
0.25	0.0
0.25	6.0
0.3	5.0
0.35	4.28
0.4	3.75
0.45	3.33
0.5	3.00
0.55	2.73
0.6	2.5
0.65	2.31
0.7	2.14
0.75	2.00
0.8	1.87
0.85	1.77
0.9	1.69
0.95	1.58
1.0	1.50
1.1	1.36
1.2	1.25
1.3	1.15
1.4	1.07
1.5	1.00
1.6	0.93
1.7	0.88
1.8	0.83
1.9	0.79
2.0	0.75

Table V.1 β as a Function of $\dfrac{\Theta_e}{\Theta_{in}}$ for the Relay with Dead Zone.
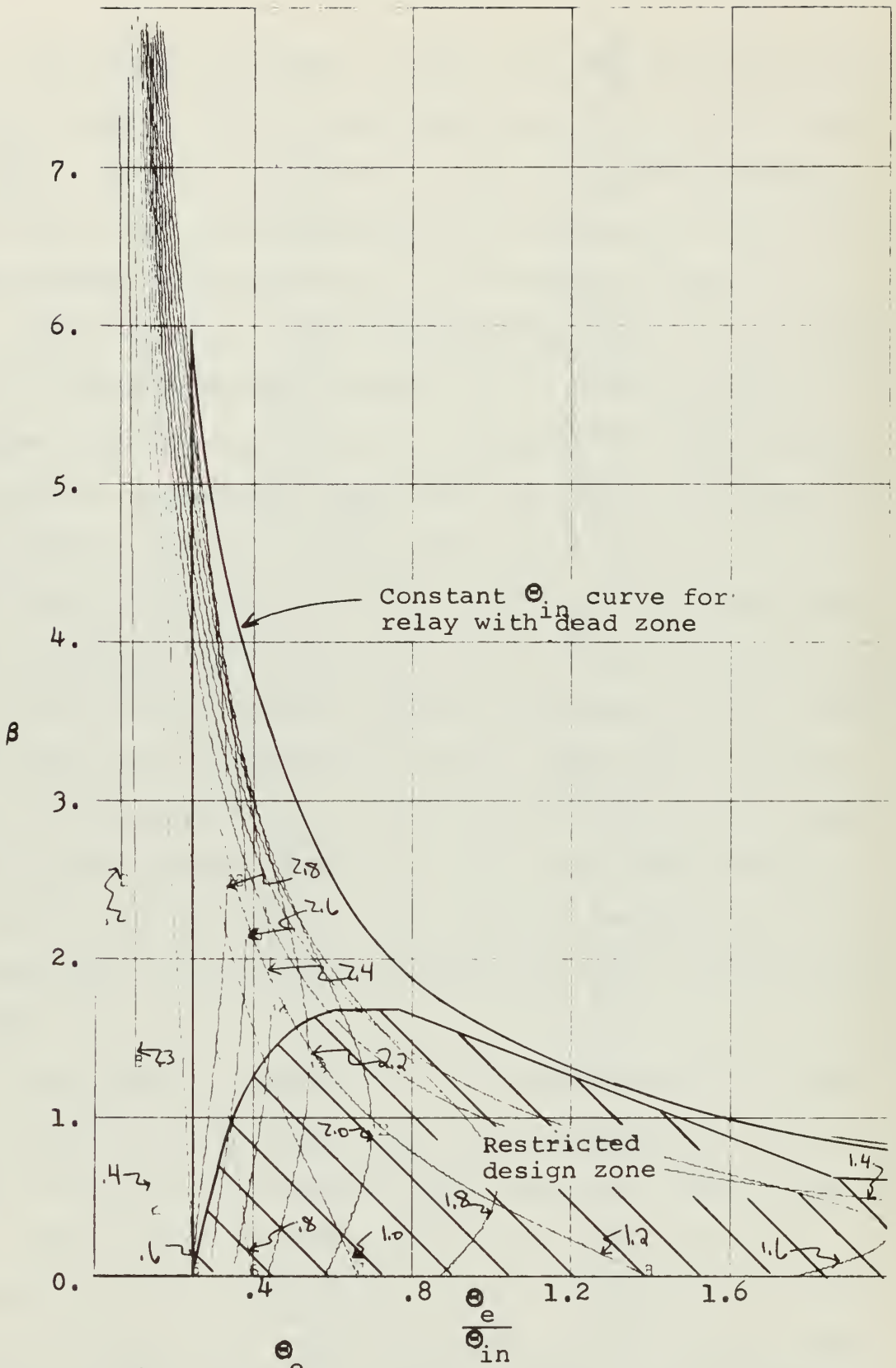
Figure 5.8 β versus $\dfrac{\Theta_e}{\Theta_{in}}$ magnitude with proposed relay with dead zone constant Θ_{in} curve
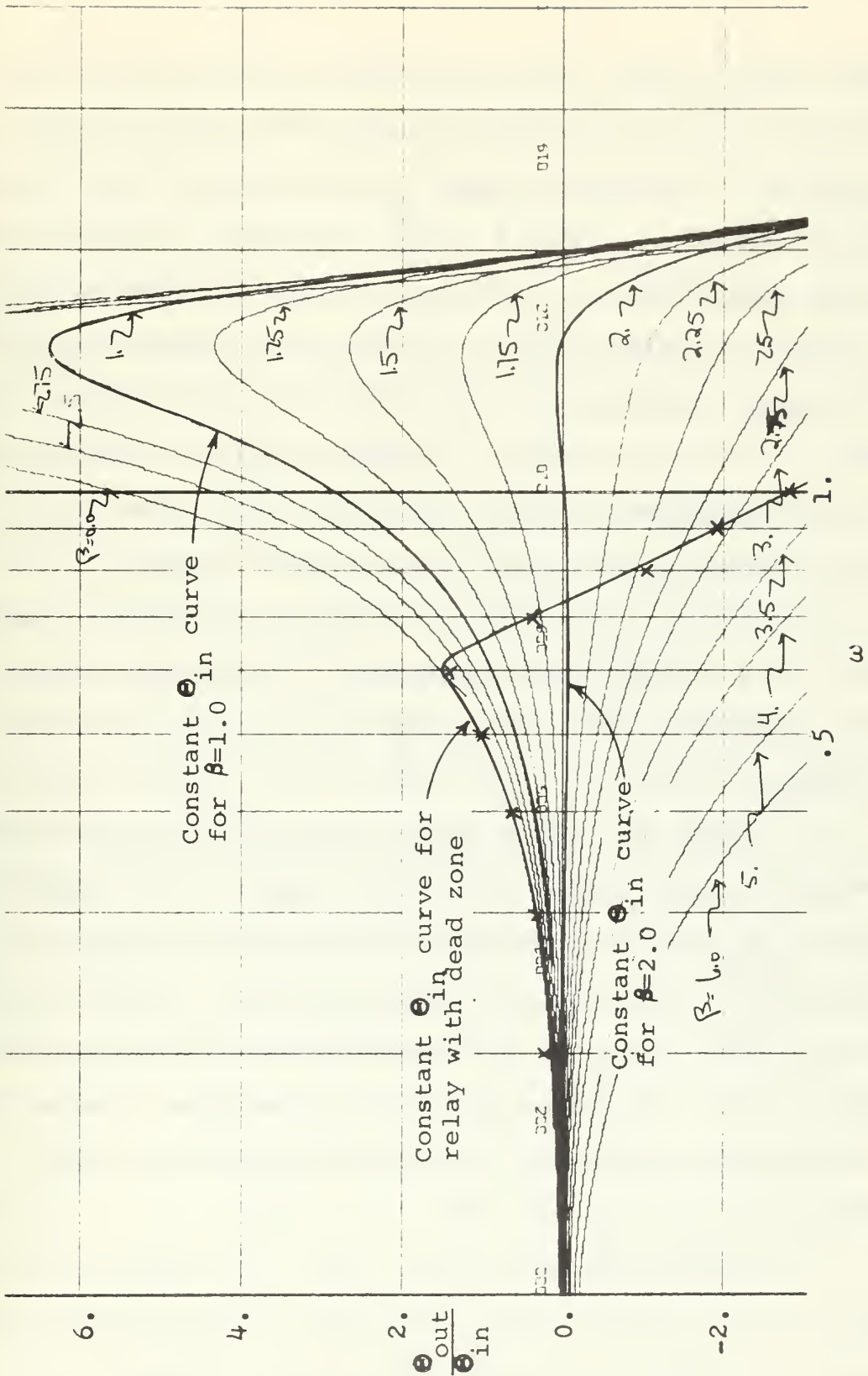
Figure 5.9 Closed loop frequency response of design
problem with proposed design solutions

137

from figure (5.8). It is seen in figure (5.9) that the
system with the relay does satisfy the conditions that
$M_p(\omega) \leq 1.2$ (1.58 dB). Also drawn on figure (5.9) is the
linear design solution, $\beta = 2.0$, which also satisfies the
design conditions. A $\beta = 1$ curve is drawn on figure (5.8)
to show the response of the system after determining K_t
but before the design of the linear/nonlinear element. The
solution has been checked by analog simulation and the re-
sults are plotted on figure (5.9) by "x's". The analog so-
lution checks closely with the graphical design.

The steps in the design are summarized below. Accord-
ing to the problem, some steps may be eliminated while
other steps may have to be added to solve the particular
problem.

1. After all linear design and compensation have been
completed, with only the nonlinear design left, obtain a
plot of β, the variable gain of the nonlinear element,
versus $\dfrac{\Theta_{out}}{\Theta_{in}}$ the magnitude, with constant ω curves drawn
on the plot. This can be accomplished by using the PARAM-5
subprogram of the PARAMS program or some other similar com-
puter program. The plot corresponding to the example in
this chapter is figure (5.5).

2. Having obtained the plot in step 1, draw on it the
area in which design cannot take place (a restricted design
zone). This zone is determined by the specifications of
the problem and must be in terms of $\dfrac{\Theta_{out}}{\Theta_{in}}$ magnitude and fre-
quency.

3. Obtain the transfer function of the signal going into the nonlinear/linear element with respect to the input signal into the system. In the problem of this chapter, this transfer function is $\frac{\Theta_e}{\Theta_{in}}$. Using the transfer function with β representing the variable gain, obtain a second plot of β versus this transfer function with the same ω curve as were drawn in the plot in step 1. This corresponds to figure (5.6) in this chapter.

4. Using the coordinates of β and ω from the plot in step 1, transfer the restricted design zone to the plot obtained in step 3. One now has a relationship for the restricted design zone in terms of β and $\frac{\Theta_e}{\Theta_{in}}$.

5. Because Θ_{in} must be specified as a constant to obtain the response of a system with a nonlinear element, the plot of step 3 is a relationship between β and Θ_e for the restricted design zone. From this relationship, a plot of Θ_a (the signal out of the linear/nonlinear element) versus Θ_e (the signal into the element) can be drawn. The restricted design zone is then drawn on this plot. This new plot corresponds to figure (5.7) in this chapter.

6. From this plot in step 5, any linear nonlinear element can be designed that does not fall into the restricted design zone. The nonlinear element must be a single-valued nonlinearity, however.

7. Make a table of β as a function of $\frac{\Theta_e}{\Theta_{in}}$ for the element that was designed. This is done by using the plot

139

in step 6, realizing that $\beta = \dfrac{\Theta_a}{\Theta_e}$ and that Θ_{in} is a constant. This corresponds to table (V.1) in this chapter.

8. Having obtained this table in step 7, plot the designed element constant Θ_{in} curve on the plot of step 5. The coordinates of the constant Θ_{in} curve are β and $\dfrac{\Theta_e}{\Theta_{in}}$ and come from the table in step 7. Having plotted the constant Θ_{in} curve on this plot, one now has the relationship between β and ω for the constant Θ_{in} curve. This relationship comes from the intersection of the designed element's constant Θ_{in} curve and the constant ω curves.

9. The final step is the reconstruction of the closed loop frequency response. The open loop frequency response can be obtained in the same manner, except by using the open loop transfer function rather than the closed loop transfer function. Obtain a plot of $\dfrac{\Theta_{out}}{\Theta_{in}}$ versus the frequency, with constant β curves. The values of β of the design element should determine what β curves are needed in this plot. Replot the constant Θ_{in} curve onto this plot using the coordinates of β and ω found in step 8. The constant Θ_{in} curve on this plot should satisfy the design specifications.

B.   SECOND DESIGN PROBLEM

In the previous analysis and design examples, the nonlinearities were functions of the signal into the element. A second type of nonlinearity is one that is a direct

function of frequency. The analysis techniques of this
thesis can be used for this second type of nonlinear ele-
ment. To illustrate how these techniques are used, a de-
sign problem is worked.

The block diagram of the second design problem is
figure (5.10). This system is the same example as was
used in chapter II. α represents the nonlinear element
which will be used to change the frequency response. From
figure (5.10), the closed loop transfer function is,

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{600\alpha}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \tag{5.15}$$

Using the equation (5.15) and the PARAM-7 subprogram,
a plot of $\frac{\Theta_{out}}{\Theta_{in}}$ (dB) versus ω is obtained. This plot is
figure (5.11) and has constant α curves plotted on it.
Also on figure (5.11) is the desired frequency response of
the system. The desired frequency response can be describ-
ed in terms of α and ω from figure (5.11).

From figure (5.10), $\frac{\Theta_e}{\Theta_{in}}$ is written,

$$\frac{\Theta_e}{\Theta_{in}} = \frac{s^4 + 18s^3 + 95s^2 + 150s}{s^4 + 18s^3 + 95s^2 + 150s + 600\alpha} \tag{5.16}$$

With equation (5.16) and the PARAM-5 subprogram, a
plot of α versus $\frac{\Theta_e}{\Theta_{in}}$ is made. This plot is figure (5.12)
and has constant ω curves drawn on it. Using the coordi-
nates of α and ω , the desired frequency response can be
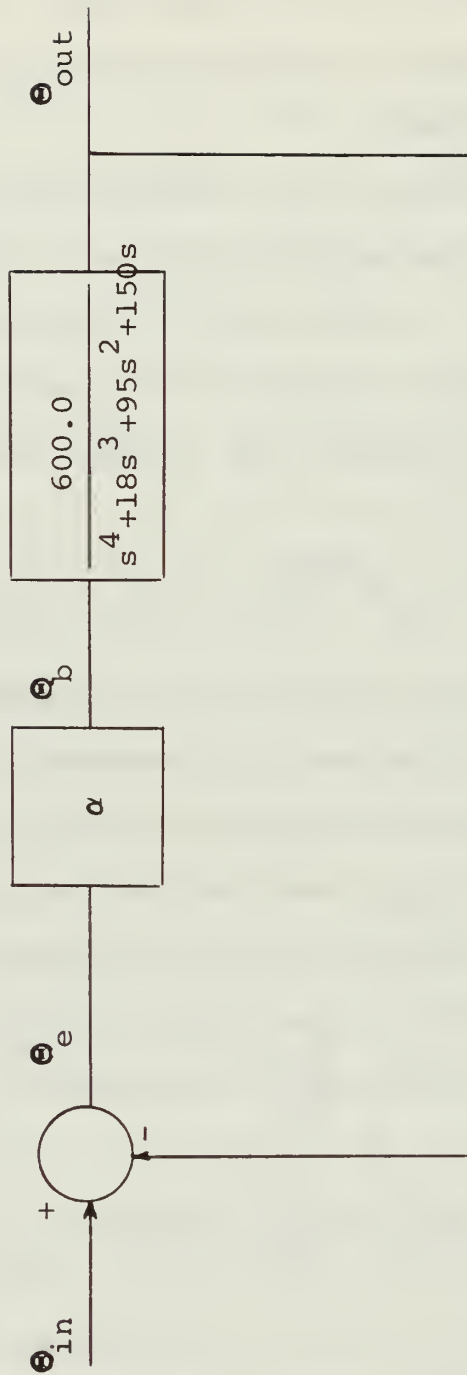
Figure 5.10 Block diagram of second design problem
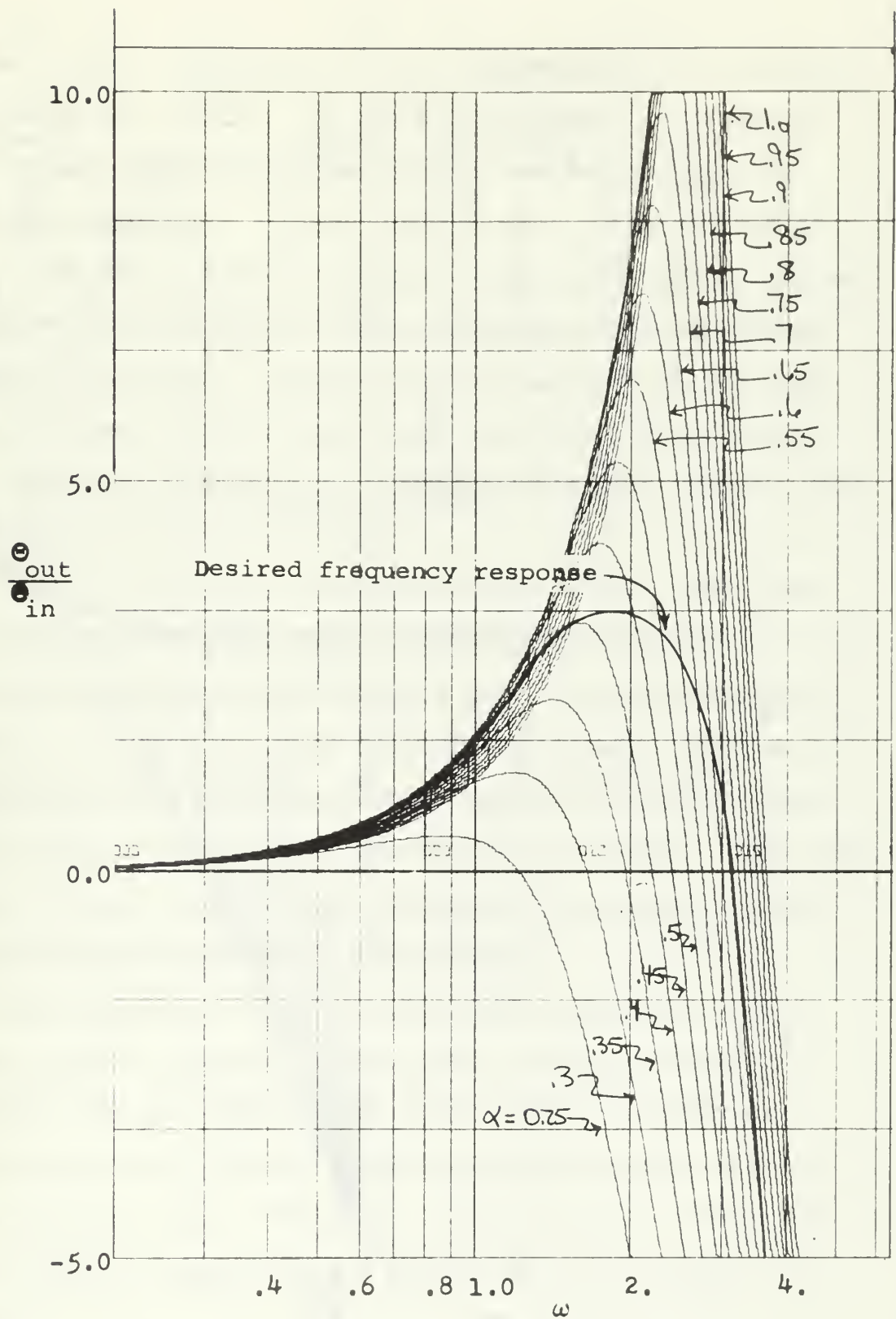
Figure 5.11 Desired closed loop frequency response of
second design problem

143
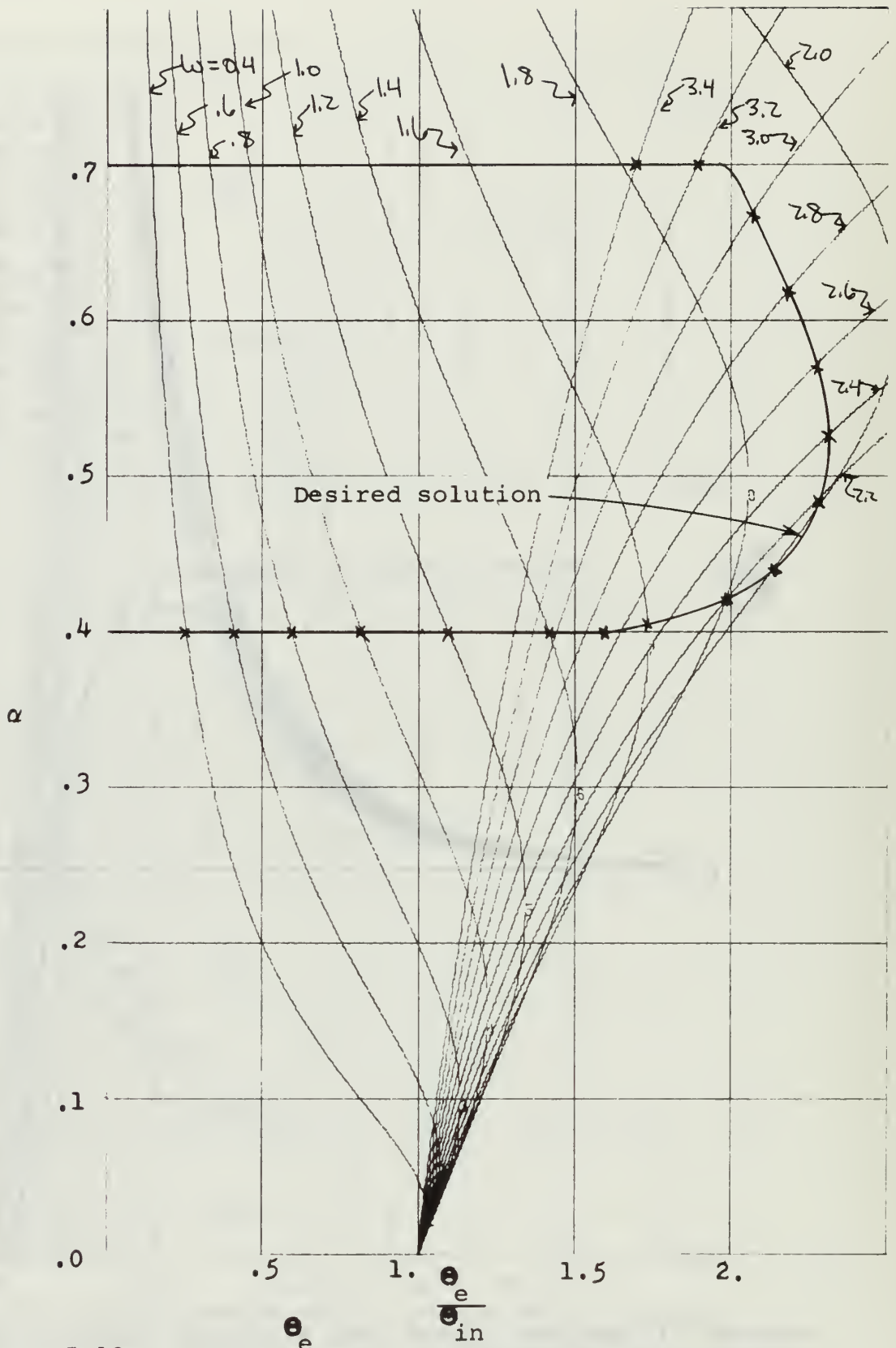
Figure 5.12 α versus $\dfrac{\theta_e}{\theta_{in}}$ for second design problem

144

replotted from figure (5.11) onto figure (5.12). As can be seen in figure (5.11), the desired solution is a function of frequency, not a function of $\frac{\Theta_e}{\Theta_{in}}$ as was the case in previous examples. If the solution were a function of $\frac{\Theta_e}{\Theta_{in}}$, then for a given frequency, there would be two values of gain in figure (5.12). This would not give the proper frequency response. Figure (5.12) gives the desired solution in terms of α and ω with $\frac{\Theta_e}{\Theta_{in}}$ as a third parameter. $\frac{\Theta_e}{\Theta_{in}}$ can only be used as a parameter when $\frac{\Theta_e}{\Theta_{in}}$ is given with an associated value of frequency.

It can be clearly seen from figure (5.12), that for $\omega \leq 1.5$, the gain, α, is 0.4. Between $\omega = 1.5$ and $\omega = 3.2$, the nonlinear device would have a gain varying bewteen 0.4 and 0.7. For $\omega \geq 3.2$, the gain of the element would be 0.7. This description of the nonlinear device can also be seen on an input-output plot of the nonlinear element. This diagram is figure (5.13) where a straight line segment represents the constant gain of the element.

The response of the system is not dependent on Θ_{in} . Thus, for this design solution, the frequency response is the same for all values of Θ_{in}. The realization of the desired nonlinear element could possibly consist of a variable amplifier with a frequency discriminator to determine what the gain of the element should be at a certain frequency.
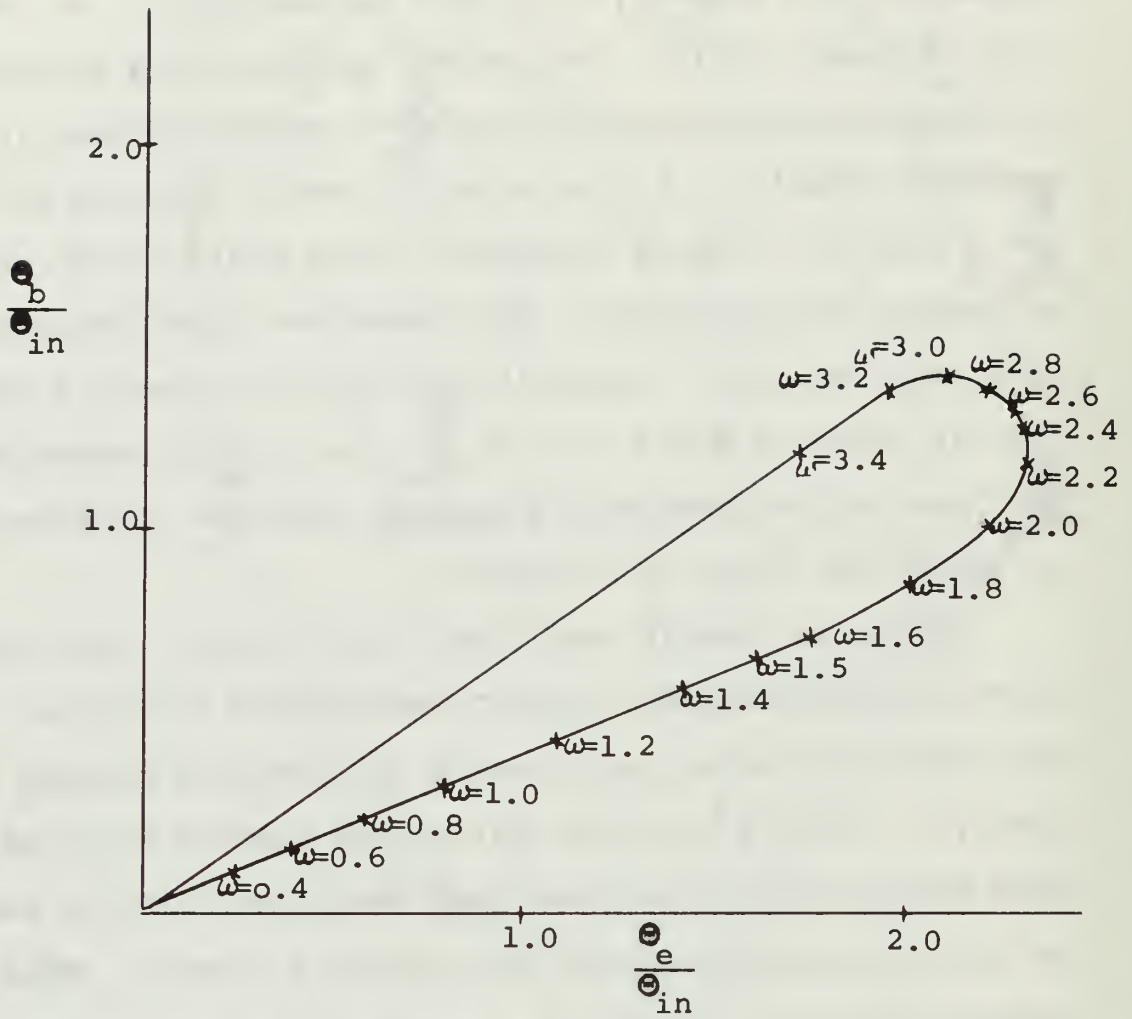
Figure 5.13 Characteristics of the nonlinear element of the
second design problem

VI. DISCUSSION, CONCLUSIONS

AND AREAS OF FUTURE STUDY

A. DISCUSSION

The accuracy of the solution of both the analysis and design depends on graphical accuracy. The plots made by the digital computer were accurate enough for the study. Any errors in the solutions came from plotting of the curves by hand and reading the data from the plots. In some cases where the scale of the plots was small and accuracy was critical, some small errors appeared in the solution.

All the examples, except for the open loop analysis, were checked by analog simulation. Appendix A explains the simulation. The nonlinear elements were simulated by using electronic switching and digital logic circuits. As a result, the nonlinear elements are ideal elements. In a real example, these elements may not be ideal, but instead show different characteristics. This presents no problem when using these analysis and design techniques as long as the nonlinear element remains single-valued. Any single-valued nonlinearity can be used in the analysis and design.

B. CONCLUSIONS

The main conclusion to be drawn from this study is that a workable, practical and reliable method has been developed to obtain the open and closed loop frequency response of

systems with a single-valued nonlinearity. This includes both magnitude and phase response. The method is fast, accurate, gives a graphical solution and involves no laborious calculations.

The method of analysis can be reversed and be used for synthesis without complicating the problem. The design techniques show clearly what are the possible design solutions, linear and nonlinear, including their exact parameters. As in the analysis, the nonlinear elements are restricted to single-valued elements. As shown in the second design problem, the analysis techniques can also be reversed to handle design solutions that are a function of only the frequency of the system. The desired frequency response remains independent of the magnitude of the signal going into the system.

The primary problem was finding the relationship between the nonlinearity and the frequency of the system. This method provides a very simple technique which permits the nonlinearity to be expressed as a variable gain and relates this gain to the frequency of the system simply and accurately.

The analysis techniques can be used for predicting jump resonance. The prediction includes not only the fact that jump resonance will occur but also the frequency of the jump resonance. This prediction can be made in the

first few steps of the analysis. The final frequency re-
sponse shows the jump resonance in both magnitude and
phase.

C.   AREAS OF FUTURE STUDY

One of the basic areas that needs investigation is in
the phase domain. With slight modification to the PARAMS
program, phase can be added as one of the variables in
plots other than just the PARAM-7 frequency response plot.

With the addition of phase, an extension of this
method may be possible so that two-valued nonlinearities
could be analyzed. These nonlinearities could include
backlash, negative deficiency, two-position relay, etc.
These types of nonlinearities could also possibly be
studied by breaking them down into real and imaginary
parts and using the two parameter capability of the PARAMS
program to handle these two parts.

Another area left unexplored is analyzing the system
with two single-valued nonlinearities. This could be de-
veloped as follows. The last step in the analysis tech-
nique was replotting the constant Θ_{in} curve to a plot of
$\frac{\Theta_{out}}{\Theta_{in}}$ versus ω to obtain the frequency response. Instead
of doing this replotting, transfer the constant Θ_{in} curve
to a plot of the nonlinear gain versus $\frac{\Theta_{out}}{\Theta_{in}}$. In the
PARAMS program, some element had to be designated as an-
other parameter value and held constant for the analysis.
By making a series of plots of nonlinear gain versus $\frac{\Theta_{out}}{\Theta_{in}}$

with the constant Θ_{in} curve, each plot having the second

parameter set to a different value, a three dimensional

surface of the constant Θ_{in} curve is developed. The coor-

dinates would be α, the nonlinear gain, β, the second pa-

rameter, $\frac{\Theta_{out}}{\Theta_{in}}$ and ω. By repeating this with a second non-

linear element, a second constant Θ_{in} surface is obtained.

The intersection of these two surfaces at a certain ω and

$\frac{\Theta_{out}}{\Theta_{in}}$ gives the closed loop frequency response of the sys-

tem with two nonlinear elements.

This technique just described would be of course very

laborious. It could be greatly simplified by developing

a digital computer program which would find the surfaces

and their intersections. This could then be extended into

a four dimensional space program to handle three nonlinear-

ities, five dimensional space to handle four nonlinearities,

etc.

APPENDIX A.   ANALOG SIMULATION

The analog simulation was performed on the
Donner 10/20 analog computer and two EAI TR-20 analog com-
puters. The Donner 10/20 was used to simulate the linear
portion of the system and the two EAI TR-20s were used to
simulate the nonlinear element.

The simulation of the linear portion of a system was
done by conventional analog simulation techniques. The
main consideration was to preserve the position of the non-
linear element in the circuit so that no accidental mani-
pulation occurred. This manipulation would correspond to
incorrectly manipulating a block diagram with a nonlinear
element. The reason for this care is the fact that the
response of a system is a function of where the nonlinear-
ity is placed in the system.

Scaling had to be used throughout the simulation and
particularly in using the Donner with the two EAI computers.
The Donner is a 100 v. machine and the EAIs are a 10 v.
machines.

The success of the total simulation rested with the
nonlinear element simulation. For this two EAI TR-20
electronic comparators were used, one electronic compara-
tor in each machine. Each electronic comparator consists
of a comparator unit which produces binary outputs and
two electronic switching units. The comparator and switches

151

have a switching time of 1.0μ sec. and the comparator has a switching sensitivity of $\pm 1.0\mu$v. The block diagram of one electronic comparator is shown in figure (A.1).[5]

The electronic comparator can be patched so that it compares two inputs, X and Y, so that,

$$X + Y \geq 0.0 \qquad E_o = E_1 \qquad \qquad \text{(A.1a)}$$

$$X + Y < 0.0 \qquad E_o = E_2 \qquad \qquad \text{(A.1b)}$$

Where E_1 and E_2 are two arbitrary inputs and E_o is the output of the electronic comparator.

One electronic comparator was used for positive signals and the other electronic comparator was used for negative signals. Two examples are worked to show how the units were used.

The first example is an ideal relay (such a figure (3.18)) where it is desired that for all positive signals into the relay, the output is 4.0 volts. For all negative signals into the relay, the output is -4.0 volts. For the first electronic comparator, X is the input signal, Y is zero, E_1 is 4.0 volts and E_2 is the input signal. Thus, when $X + Y \geq 0.0$, which is when the input signal is greater than zero, the output from the first electronic comparator is E_1 or 4.0 volts. When $X + Y < 0.0$, which is

---

5
    Electronic Associates, Inc.   TR-20 Computer Opera-
tor's Reference Handbook, 1964.
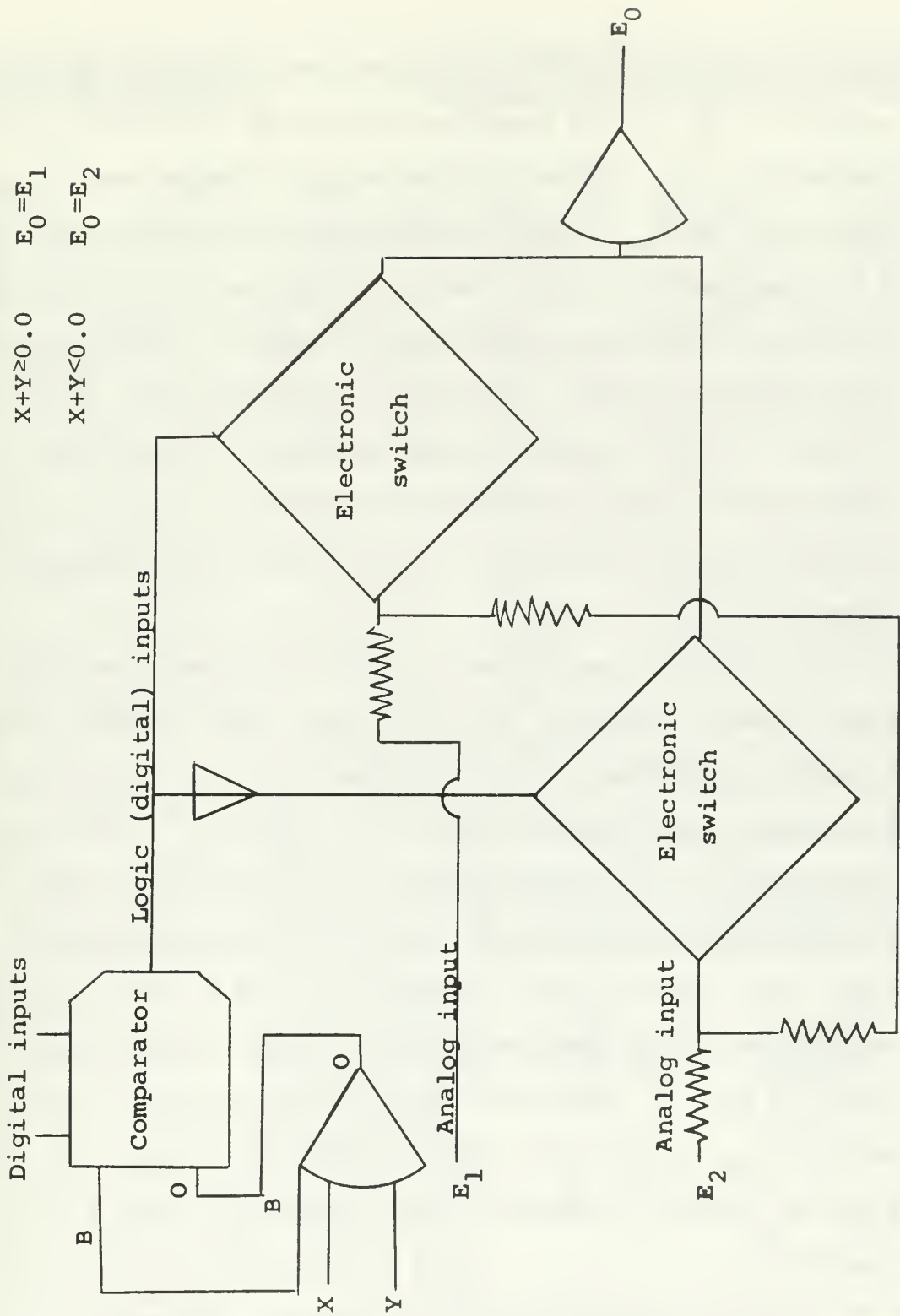
Figure A.1   EAI TR-20 electronic comparator

153

when the input signal is negative, the output of the comparator is E_2 or the negative portion of the input signal. The output, E_o, of the first electronic comparator is made the X and the E_1 of the second electronic comparator. The Y is zero and E_2 is -4.0 volts. Thus when the output of the first electronic comparator is negative, the output of the second electronic comparator is -4.0 volts. When the output of the first electronic comparator is positive, the output of the second electronic comparator is +4.0 volts which is the only positive output of the first electronic comparator.

The second example is a dead zone element whose dead space extends ± 4.0 v. Let the input into the dead zone element be represented by the symbol IN. For the first electronic comparator X = IN, Y = 4.0 v., E_1 = IN - 4. and E_2 = 0.0. Thus for positive signals greater than +4.0 v., the output, E_{01}, of the first electronic comparator is IN-4. For signals less than +4., E_{01} = 0.0. For the second electronic comparator let X = IN, Y = +4., E_1 = E_{01} and E_2 = IN + 4. Thus for signals less than -4., the output, E_{02}, is IN + 4. For signals greater than -4., E_{02} = E_{01} which is zero for -4.0 ≤ IN ≤ 4.0 and IN - 4. for IN > 4.

With the switching time of 1.0 μsec. and sensitivity of 1μv., the response is very good. The output of the second electronic comparator is fed back into the linear system with appropriate scaling.

```
C     PROGRAM PARAMS                                                    0003
C     THIS PROGRAM CONVERTS A GIVEN TRANSFER FUNCTION TO A SET OF       0004
C     DOMAIN REPRESENTATIONS FOR ANALYSIS OR DESIGN STUDIES. THIS       0005
C     FUNCTION IS READ AS A RATIO OF TWO FREQUENCY POLYNOMIALS WITH     0006
C     TWO VARIABLE PARAMETERS, ALPHA AND BETA, EMBEDDED IN THE          0007
C     THE PROGRAM THEN TRANSFORMS THIS FUNCTION INTO THE ANGULAR        0008
C     FREQUENCY DOMAIN, YIELDING AN EQUATION IN TERMS OF FOUR VARIABLES:0009
C     ALPHA, BETA, MAGNITUDE, AND OMEGA. FIXING ONE TWO-DIMENSIONAL     0010
C     REPRESENTATIONS ARE DESIRED CURVE VALUES BY FIXING ONE VARIABLE   0011
C     TWO VARIABLES ARE GENERATED BY AND LETTING THE THIRD POLYNOMIAL.  0012
C     ASSIGNING DESIRED CURVE VALUES TO THE ORDINATE AS A CONSTANT:     0013
C     VARIABLE PARAMETER, GAMMA, THE ABOVE VALUES INTO EACH VALUE OF THE0014
C     IN THIS CASE, THE ABOVE PROCESS IS REPEATED FOR EACH VALUE OF THE 0015
C     VARIABLE. DATA CARD INFORMATION FOLLOWS                           0016
*     PROGRAM MODIFICATION CARD:                                        0017
*     DUE TO ADDITIONAL CREQUIREMENTS, PARAM-7 HAS BEEN MODIFIED.       0018
*     IN PARAM-7, ONLY THE MAG. AND PHASE COMBINATIONS ARE ALLOWABLE.   0019
*     PHASE, MAG. AND GAMMA. LIMITS MUST BE TAKEN ON LIMITS OF          0020
*     XMAGMN AND XMAGMX (X,PARAMS) TOTAL MEMORY REQUIREMENT             0021
*     IS 240K FOR ALL 7-PARAM. IF REPLACED BY A DUMMY CARD:             0022
*     0143 IN THE DIMENSION STATEMENT. V(1,1) ARE TO BE USED. CARD 0276 0023
*     (DO 31), T(1,1), U(1,1), W(1,1) MEMORY REQUIREMENT IS DECREASED   0024
*     DATA CARD INFORMATION SECTION                                     0025
C     SYMBOL                                                            0026
C     NRUN     NUMBER OF DESIRED RUNS. EACH MUST HAVE A COMPLETE        0027
C              SET OF DATA CARDS LISTED BELOW. NOT EVERY SYMBOL         0028
C              IS USED IN ALL CASES. LEAVES UNUSED SYMBOLS BLANK.       0029
C              DO NOT INSERT ADDITIONAL BLANK CARDS SO SPECIFIED.       0030
C     KPARAM   NUMBER OF GRAPHICAL PRESENTATIONS OF EACH PARAM ABOVE.   0031
C     JPARAM   NUMBER OF NUMERICAL TITLES (1 TO 7) WITH KPARAM TO BE RUN:0032
C              MUST AGREE IN NUMBER.                                    0033
C     NORN     ORDER OF NUMERATOR POLYNOMIAL (LIMITED TO 49)            0034
C     NORD     ORDER OF DENOMINATOR POLYNOMIAL (LIMITED TO 49)          0035
C     MODORD   Y-AXIS VARIABLE. IS ORDINATE VARIABLE CONTROL            0036
C              MODORD=1: IS ORDINATE VARIABLE (PARAM 1-6)               0037
C              MODORD=2: ALPHA A CURVES (PARAM 1-6)                     0038
C              MODORD=2: BETA A CURVES (PARAM 7)                        0039
C     NALPHA   NUMBER OF ALPHA CURVES DESIRED (LIMITED TO 16)           0040
C     NBETA    NUMBER OF BETA CURVES DESIRED (LIMITED TO 16)            0041
C     NMAG     NUMBER OF MAGNITUDE CURVES DESIRED (LIMITED TO 16)
C     NOMEGA   NUMBER OF OMEGA CURVES DESIRED (LIMITED TO 16)
C     NGAMMA   POSITIVE INTEGER DENOTES NUMBER OF THIRD PARAMETER
C              VALUES TO BE EMBEDDED INTO EACH RUN. INSERT
C              COEFFICIENTS FOR THESE GAMMA VALUES ON SEPARATE
C              CARDS DIRECTLY BEHIND REST OF COEFFICIENTS; SEE #19

      CARD   FORMAT   SYMBOL
      1      8I10     NRUN

      2      8I10     KPARAM
                      JPARAM

      3      8I10     NORN
                      NORD
                      MODORD

                      NALPHA
                      NBETA
                      NMAG
                      NOMEGA
                      NGAMMA
```

155

```
0042  C      IMAG      NEGATIVE INTEGER DENOTES NUMBER OF PARAM-7 VECTOR
0043  C                ADDITIONS (NEGATIVE) FOR GENERALIZED BLOCK DIAGRAM STUDIES.
0044  C                THE VALUE OF ENGAMMA DETERMINES HOW MANY
0045  C                BODE CALCULATIONS WILL BE MADE BEFORE PLOTTING THE
0046  C                VECTOR RESULTANT AS ONE VALUE OF MAGNITUDE; AND ONE PHASE.
0047  C                INSERT ADDITIONAL SETS OF COEFFICIENTS AT END.
0048  C                MAGNITUDE=0: ALL VALUES IN DECIBELS (PARAM 7 THIS FORM)
0049  C                IMAG=1: MEANS ALL VALUES IN ACTUAL MAGNITUDE
0050  C                IMAG=2 MEANS ALL VALUES IN MAGNITUDE SQUARED
0051  C
0052  C      IWRITE    PRINTOUT CONTROL
0053  C                IWRITE=-1 MEANS ONLY PRINTOUT INPUT DATA
0054  C                IWRITE=0 MEANS INPUT DATA WILL BE PRINTED
0055  C                IWRITE=1 TO 900 YIELDS INPUT DATA;GRAPHICAL
0056  C                INFORMATION, AND PRINTOUT. EACH IWRITE IS DICTATED THE NUMBER
0057  C                OF COMPUTATIONS. RESULTS. THE NUMBER
0058  C                BY THE VALUE OF IWRITE. IF IWRITE IS GREATER THAN
0059  C                900 ONLY INPUT DATA & GRAPH INFO. WILL BE PLOTTED
0060  C
0061  C      IGRAPH    FREQUENCY SCALING CONTROL
0062  C                IGRAPH=-1: NO GRAPHS WILL BE PLOTTED.
0063  C                IGRAPH=0: ONLY PARAMS 7 WILL BE PLOTTED ON LOG GRID
0064  C                IGRAPH=1: PARAMS 5&7 WILL BE PLOTTED ON LOG GRID
0065  C                IGRAPH=2: PARAMS 4,6,7 WILL BF PLOTTED ON LOG GRID.
0066  C
0067  C      IPHASE    PHASE PLOT CONTROL
0068  C                PARAM 7 PHASE CURVES
0069  C                IPHASE=-1 PHASE CURVES SUPERIMPOSED ON MAGNITUDE
0070  C                IPHASE=0: PHASE CURVES PLOTTED ON SEPARATE GRAPH
0071A C               IPHASE=1 PHASE PLOTTED ON SEPARATE GRAPH
0072A C
0072A C      IQUAD     PHASE CURVE SCALING CONTROL
0073  C                PARAM 7 PHASE CURVES AUTOMATICALLY SCALED AT 45
0074  C                IQUAD=0: DEGREES/INCH WITH -180 COINCIDENT WITH 0 DB MAG.
0075  C                IQUAD=1 TO 100: PHASE CURVES FROM 0 DEGREES TO
0076  C                -(IQUAD) DEGREES/INCH. SCALE OF IQUAD DEG/INCH
0077  C
0078  C      IPCLAR    PHASE PLOT CONTROL
0079  C                PARAM 7 POLAR PLOT ON SEPARATE GRAPH
0080  C                IPOLAR=0: MEANS NO POLAR PLOT
0081  C                IPOLAR=1 GIVES POLAR PLOT
0082  C      LX        LENGTH OF X-AXIS (LIMITED TO 15 INCHES)
0083  C      LY        LENGTH OF Y-AXIS (LIMITED TO 9 INCHES)
0084  C      CONALP    VALUE OF ALPHA WHEN USED AS A CONSTANT
0085  C      CONBET    VALUE OF BETA WHEN USED AS A CONSTANT
0086  C      CONCAE    VALUE OF OMEGA WHEN USED AS A CONSTANT; SEE IMAG (NOT ZERO)
0087  C *****DATA CARD INFORMATION SECTION (CONTINUED)*****
      C      SYMBOL    USAGE OF SYMBOL
      C      ALPHA     MIN. VALUE OF ALPHA WHEN USED AS AN AXIS VARIABLE
      C      ALPHMX    MAX. VALUE OF ALPHA WHEN USED AS AN AXIS VARIABLE
      C      BETA      MIN. VALUE OF BETA WHEN USED AS AN AXIS VARIABLE
      C      BETAMX    MAX. VALUE OF BETA WHEN USED AS AN AXIS VARIABLE
```

```
4     8I10

5     8F10.3

      *****
      *****
      *****
      *****  CARD FORMAT
6     8F10.3
      *****
      *****
      *****
```

```
0088  C     XMAGMN                 7   MIN. MAG. VALUE WHEN USED AS AXIS VAR.:: SEE IMAG
0089  C     XMAGMX                 8   MAX. MAG. VALUE WHEN USED AS AXIS VAR.:  SEE IMAG
0090  C     OMEGMN                     MIN. OMEGA VALUE WHEN USED AS AXIS VARIABLE
0091  C     OMEGMX                     MAX. OMEGA VALUE WHEN USED AS AXIS VARIABLE (SEE IGRAPH), OMEGMN
0092  C                                NOTE: IF LOG GRID IS REQUESTED TO NEAREST POWER OF 10, AND
0093  C                                WILL BE AUTO. CHANGED.THE NEAREST POWER OF 10, OMEGMN
0094  C                                OMEGMX CHANGED LIKEWISE.THE NUMBER OF DECADES IS
0095  C                                AUTOMATICALLY COMPUTED FROM THESE NEW VALUES.
0096  C     ZALPHA    8F10.3        9   DESIRED VALUES FOR ALPHA CURVES (SEE NALPHA)
0097  C     ZBETA     8F10.3       10   DESIRED VALUES FOR BETA CURVES (SEE NBETA)
0098  C     ZMAG      8F10.3       11   DESIRED MAGNITUDE CURVE VALUES (SEE NMAG&IMAG)
0099  C     ZOMEGA    8F10.3       12   DESIRED POSITIVE OMEGA CURVE VALUES (SEE NOMEGA)
0100  C     HAT       8F10.3       13   NUMERATOR CONSTANT COEFFICIENTS IN ASCENDING ORDER
0101  C     HBT       8F10.3       14   NUMERATOR ALPHA COEFFICIENTS IN ASCENDING ORDER
0102  C     HCT       8F10.3       15   NUMERATOR BETA COEFFICIENTS IN ASCENDING ORDER
0103  C     HDT       8F10.3       16   DENOMINATOR CONST. COEFFICIENTS IN ASCENDING ORDER
0104  C     HET       8F10.3       17   DENOMINATOR ALPHA COEFFICIENTS IN ASCENDING ORDER
0105  C     HFT       8F10.3       18   DENOMINATOR BETA COEFFICIENTS IN ASCENDING ORDER
0106  C                          *      DENOM ALPHA-BETA COEFFICIENTS IN ASCENDING ORDER
0107  C                                 OMIT CARDS #19 THRU #27 IF NGAMMA LESS THAN ONE.
0108  C     ZGAMMA    8F10.3       19   VALUES OF THE THIRD PARAMETER: MUST MATCH NGAMMA
0109  C     GAMHAT    8F10.3       20   NUM. GAMMA-CONST. COEFFICIENTS IN ASCENDING ORDER
0110  C     GAMHBT    8F10.3       21   NUM. GAMMA-ALPHA COEFFICIENTS IN ASCENDING ORDER
0111  C     GAMHCT    8F10.3       22   NUM. GAMMA-BETA COEFFICIENTS IN ASCENDING ORDER
0112  C     GAMHDT    8F10.3       23   DEN. GAMMA-CONST. COEFFICIENTS IN ASCENDING ORDER
0113  C     GAMHET    8F10.3       24   DEN. GAMMA-ALPHA COEFFICIENTS IN ASCENDING ORDER
0114  C     GAMHFT    8F10.3       25   DEN. GAMMA-BETA COEFFICIENTS IN ASCENDING ORDER
0115  C                                 DEN. GAMMA-ALPHA-BETA COEFFICIENTS IN ASCENDING ORDER
0116  C     ITITLE    6A8          26   FIRST LINE OF GRAPH TITLE (COLUMNS 1 THRU 48)
0117  C     ITITLE    6A8          27   SECOND LINE OF GRAPH TITLE (COLUMNS 1 THRU 48)
0118  C                                 THE NUMBER OF GRAPHS REQUESTED FOR PARAM.(SEE IPOLAR)
0119  C     JTITLE    6A8          30   1ST PHASE TITLE: OMIT UNLESS NPARAM=7&IPHASE=1
0120  C     JTITLE    6A8          31   2ND PHASE TITLE: OMIT UNLESS NPARAM=7&IPHASE=1
0121  C     KTITLE    6A8          32   1ST POLAR TITLE: OMIT UNLESS NPARAM=7&IPOLAR=1
0122  C     KTITLE    6A8          33   2ND POLAR TITLE: OMIT UNLESS NPARAM=7&IPOLAR=1
0123  C     ***********************************************************************
0124  C     ***  E(TITLE(12),JTITLE(12),KTITLE(12),JOKER(9)/1H ,6HPARAM-,7HFUR
0125  C     REAL*8ITITLE,8HMAGNITUD,5HGAMMA,8HSEPARATE/
0126  C     REAL*8ALPHA,4HBETA,5HOMEGA,4H   ,7HIPHASE ,6HIPOLAR)
0127  C     REAL DM/1H1/,LA(66)/4H 0.1,4H 0.2,4H 0.3,4H 0.4,4H 0.5,4H 0.6,4H 0
0128  C     E.7,4H 0.8,4H 0.9,4H1.0 ,4H2.0 ,4H3.0 ,4H4.0 ,4H5.0 ,4H 6.0,
0129  C     E4H 70.,4H 80.,4H 90.,4H100.,4H200.,4H300.,4H400.,
0130  C     E4H600.,4H700.,4H800.,4H900.,4H1000,4H2000,4H3000,
0131  C     E4H500.,4H600.,4H700.,4H800.,4H900.,4H1000,1H ,1H1,1H2,1H3,1H4,1H5,
0132  C     E4H4000,4H5000,4H6000,4H7000,4H8000,4H9000,1H ,1H1,1H2,1H3,1H4,1H5,
0133
0134
0135  C     ***********************************************************************
```

```
      E1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13,2H14,2H15,2H16/
      DIMENSICN JPARAM(8),A(50),B(50),C(50),D(50),E(50),HA(50),HAT(50),HBT(50)
     E,HCT(50),HDT(50),HD(50),HET(50),OLDHA(50),OLDHAT(50),OLDHB
     ET(50),OLDHCT(50),HFT(50),OLDHD(50),OLDHET(50),OLDHFT(50),GAMHA(
     E50),GAMHBT(50),GAMHAT(50),GAMHCT(50),GAMHD(50),GAMHDT(50),GAMHET(5
     E0),GAMHFT(50),IC(16),ZALPHA(16),ZBETA(16),ZGAMMA(16),ZMAGSQ(16),ZO
     ES(16,900),T(16,900),U(16,900),V(16,900),W(16,900)
      EQUIVALENCE(U,W,S),(V,T)
      CALL CANCEL(2)
    1 FORMAT(3I10)
    2 FORMAT(10X,8I10)
    3 FORMAT(6A8)
    4 FORMAT(10X,6A8)
    5 FORMAT(8F10.3)
    6 FORMAT(10X,1P10E12.3)
C ***** INPUT DATA READ IN *********************************
      READ(5,2) NRUN
      DO 1000 IRUN=1,NRUN
      READ(5,1) KPARAM,(JPARAM(I),I=1,KPARAM)
      READ(5,1) NORD,MODORD,NALPHA,NBETA,NMAG,NOMEGA,NGAMMA
      READ(5,1) NIMAG,IWRITE,IGRAPH,ICONJME
      READ(5,5) CONALP,CONMAG,CONOMET,CONJMX,BETAMN,ALPHA)
      READ(5,5) ALPHMN,ALPHMX,BETAMX,XMAGMN,XMAGMX,OMEGMN,OMEGMX
      READ(5,5) (ZALPHA(I),I=1,NALPHA)
      READ(5,5) (ZBETA(I),I=1,NBETA)
      READ(5,5) (ZMAGSQ(I),I=1,NMAG)
      READ(5,5) (ZOMEGA(I),I=1,NOMEGA)
      LOE=NCRA+1
      MOE=NCRD+1
      READ(5,5) (ZGAMMA(I),I=1,NGAMMA)
      READ(5,5) (GAMHA(I),I=1,LOE)
      READ(5,5) (GAMHAT(I),I=1,LOE)
      READ(5,5) (GAMHBT(I),I=1,LOE)
      READ(5,5) (GAMHCT(I),I=1,MOE)
      READ(5,5) (GAMHDT(I),I=1,MOE)
      READ(5,5) (GAMHET(I),I=1,MOE)
      READ(5,5) (GAMHFT(I),I=1,MOE)
      IF((HAT(I).GE.NGAMMA)) (ZGAMMA(I),I=1,NGAMMA)
      IF((HAT(I).GE.NGAMMA)) (GAMHA(I),I=1,LOE)
      IF((HBT(I).GE.NGAMMA)) (GAMHBT(I),I=1,LOE)
      IF((HCT(I).GE.NGAMMA)) (GAMHCT(I),I=1,LOE)
      IF((HDT(I).GE.NGAMMA)) (GAMHDT(I),I=1,MOE)
      IF((HET(I).GE.NGAMMA)) (GAMHET(I),I=1,MOE)
      IF((HFT(I).GE.NGAMMA)) (GAMHFT(I),I=1,MOE)
      IF((IWRITE).LT.(0)) GO TO 24
```

```
C******************INPUT DATA PRINT OUT SECTION***************************
******  WRITE(6,7) IRUN,NORN,NORD,KPARAM,(JOKER(2),JPARAM(I),I=1,KPARAM)
    7 FORMAT(1H1,10X,'****  THE INPUT DATA FOR RUN',I2,' IS',*****'.//,
     €10X,'ORDER OF NUMERATOR:',I2,//,10X,'ORDER OF DENOMINATOR:',I2,//,
     €10X,'THE FOLLOWING',I2,' GRAPHICAL CONFIGURATIONS WILL BE COMPUTED
     €',//,20X,8(A8,I1,3X),/)
      IF((NGAMMA).GE.(1)) WRITE(6,8) NGAMMA,(ZGAMMA(I),I=1,NGAMMA)
    8 FORMAT(10X,I2,' GAMMA OR THIRD VAR.PARAMETER COEFFICIENTS WILL B
     €E USED.',//,10X,'THE GAMMA VALUES ARE:',1P8E12.3,/,10X,1P8E12.3,/)
      WRITE(6,10)
   10 FORMAT(/,10X,'THE COEFFICIENTS FOR THE NUMERATOR ARE:',/)
      DO 11 I=1,LCE
      J=I-1
      IF(NGAMMA .LE.0) WRITE(6,12) HA(I),HAT(I),HBT(I),HCT(I),J
   12 FORMAT(20X,1PE10.3,'+',1PE10.3,'*ALPHA*BETA + ',1PE10.3,'*BETA + '
     €,1PE10.3,'**ALPHA + S**(',I2,')')
      IF(NGAMMA .GT.0) WRITE(6,9) HA(I),GAMHA(I),HAT(I),GAMHAT(I),HBT(I
     €),GAMHBT(I),HCT(I),GAMHCT(I),J
    9 FORMAT(15X,1PE10.3,'+(',1PE10.3,'*GAMMA)+(',1PE10.3,'+',1PE10.
     €3,'*GAMMA)*ALPHA+(',1PE10.3,'+',1PE10.3,'*GAMMA)*BE
     €TA+(',1PE10.3,'+',1PE10.3,'*GAMMA)*ALPHA*BETA)**(',I2,')',/)
   11 CONTINUE
      WRITE(6,13)
   13 FORMAT(/,10X,'THE COEFFICIENTS FOR THE DENOMINATOR ARE:',/)
      DO 14 I=1,MOE
      J=I-1
      IF(NGAMMA .LE.0) WRITE(6,12) HD(I),HDT(I),HCT(I),HET(I),HFT(I),J
      IF(NGAMMA .GT.0) WRITE(6,9) HD(I),GAMHD(I),HDT(I),GAMHDT(I),HET(I
     €),GAMHET(I),HFT(I),GAMHFT(I),J
   14 CONTINUE
      WRITE(6,15)
   15 FORMAT(15X,'     NORN     NORD     MODORD     NALPHA     NBETA     NMA
     €G     NOMEGA     NGAMMA',/)
      WRITE(6,16) NORN,NORD,MODORD,NALPHA,NBETA,NMAG,NOMEGA,NGAMMA
   16 FORMAT(/,17X,'IMAG     IGRAPH     IPHASE     IQUAD     IPO
     €LAR     LXAXIS     LYAXIS     IWRITE',/)
      WRITE(6,17) IMAG,IWRITE,IGRAPH,IPHASE,IQUAD,IPOLAR,LX,LY
   17 FORMAT(/,12X,'  CCNALPHA     CCNBETA     CONMAG     CONOMEGA ',/)
      WRITE(6,6)CONALP,CONBET,CONMAG,CONOME
   18 FORMAT(/,12X,'  ALPHMIN     ALPHMAX     BETAMIN     BETAMAX     XMAG
     €MIN     XMAGMAX     OMEGAMIN     OMEGAMAX .',/)
      WRITE(6,6)ALPHMN,ALPHMX,BETAMN,BETAMX,XMAGMN,XMAGMX,OMEGMN,OMEGMX
      WRITE(6,19) (ZALPHA(I),I=1,NALPHA)
      WRITE(6,19) (ZBETA(I),I=1,NBETA)
      WRITE(6,19) (ZMAGSQ(I),I=1,NMAG)
```

159

```
      WRITE(6,19) JCKER(6),(ZOMEGA(I),I=1,NOMEGA)                        0232
   19 FORMAT(/,10X,A8,'CURVE VALUES:',1P8E12.3,/31X,1P8E12.3)            0233
      IF(IWRITE) 20,20,22                                               0234
      WRITE(6,21)                                                       0235
   20 FORMAT(/,10X,'THERE WILL BE NO PRINTOUT OF COMPUTED RESULTS')     0236
      GO TO 24                                                          0237
   22 WRITE(6,23) IWRITE                                                0238
   23 FORMAT(/,10X,'COMPUTED RESULTS WILL BE PRINTED EVERY',I3,' STEPS')0239
                                                                        0240
C******************INITIALIZING COMPUTATIONAL SETTINGS****************  0241
   24 NPAR7=IABS(NGAMMA)                                                0242
      IF(MCDORD.GE.2) GO TO 25                                          0243
      DUMMY1=BETAMN                                                     0244
      DUMMY2=BETAMX                                                     0245
      BETAMN=ALPHMN                                                     0246
      BETAMX=ALPHMX                                                     0247
      ALPHMN=DUMMY1                                                     0248
      ALPHMX=DUMMY2                                                     0249
   25 DO 100 IPARAM=1,KPARAM                                            0250
      NPARAM=JPARAM(IPARAM)                                             0251
      MGAMMA=NGAMMA+1                                                   0252
      DO 100 IGAMMA=1,MGAMMA                                            0253
      GAMMA=ZGAMMA(IGAMMA)                                              0254
      IF(NGAMMA.GE.1) GO TO 517                                         0255
      XS=((ALPHMX-ALPHMN)/LX                                           0256
      IF((ABS(XS)).LE.(1.0E-10)) XS=1.0E-10                            0257
      YS=((BETAMX-BETAMN)/LY                                           0258
      IF((ABS(YS)).LE.(1.0E-10)) YS=1.0E-10                            0259
      IX=-BETAMN/YS                                                     0260
      IY=-ALPHMN/XS                                                     0261
      MOD=1                                                             0262
      JCURVE=0                                                          0263
      IPAR7=1                                                           0264
      READ(5,3) (ITITLE(I),I=1,12)                                      0265
      IF(NPARAM.NE.7)GO TO 34                                           0266
      IF(IMAG-1)27,28,29                                                0267
   27 DBMAX=XMAGMX                                                      0268
      DBMIN=XMAGMN                                                      0269
      GO TO 30                                                          0270
   28 DBMAX=20.0*ALOG10(XMAGMX)                                         0271
      DBMIN=20.0*ALOG10(XMAGMN)                                         0272
      GO TO 30                                                          0273
   29 DBMAX=10.0*ALOG10(XMAGMX)                                         0274
      DBMIN=10.0*ALOG10(XMAGMN)                                         0275
   30 XS=(DBMAX-DBMIN)/LX                                              0276
      XX=DBMAX                                                          0277
      IY=DBMAX/(20.0*.4342945))
      DO 31 I=1,16
      S(I,1)=0.0
```

```
      T(I,1)=0.0
      U(I,1)=0.00
      V(I,1)=0.00
   31 W(I,1)=0.0
      IF(IPHASE.LE.0) GO TO 32
      READ(5,3)(JTITLE(I),I=1,12)
      IF((IPOLAR).LE.(0)) GO TO 33
      READ(5,3)(KTITLE(I),I=1,12)
   32 IF(MODORD-2)45,44,44
   33 GO TO (35,40,41,46,47,51,59,59),NPARAM
   34 NLIMIT=NCMEGA
   35 IF(IMAG-1)37,38,39
   36 DUMMY1=10.0**(CONMAG/10.0)
   37 IF(NPARAM-1)59,59,43
   38 DUMMY1=CONMAG*CONMAG
      IF(NPARAM-1)59,59,43
   39 DUMMY1=CONMAG
      IF(NPARAM-1)59,59,43
   40 NLIMIT=NMAG
   41 OMEGA=CONCME
      IF(NPARAM-2)59,59,42
   42 XS=(XMAGMX-XMAGMN)/LX
      IY=-XMAGMN/XS
   43 IF(MODORD-2)44,45,45
   44 NLIMIT=NBETA
      IF(NPARAM-6)59,53,47
   45 NLIMIT=NALPHA
      IF(NPARAM-6)59,53,47
   46 NLIMIT=NMAG
      XS=(OMEGMX-CMEGMN)/LX
      IY=-COMEGMN/XS
   47 IF(MODORD-2)48,49,49
   48 CONST=CONBET
      IF(NPARAM-4)52,52,50
   49 CONST=CCNALP
      IF(NPARAM-4)52,52,50
   50 IF(NPARAM.GE.7) GO TO 55
      XS=(XMAGMX-XMAGMN)/XS
      IY=-XMAGMN/XS
      NLIMIT=NCMEGA
      GO TO 59
   51 XS=(OMEGMX-OMEGMN)/LX
      IY=-COMEGMN/XS
      GO TO 36
   52 IF(IGRAPH)59,59,54
   53 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 59
   54 IY=LX-IX
   55 IF(IGRAPH.LE.-2) GO TO 59
```

```
C*********LCG GRID GRAPHICAL CONSTRUCTION***************************
      IF(OMEGMN.LT.1000.)YLOGMN=100.
      IF(OMEGMN.LT.100.)YLOGMN=10.
      IF(OMEGMN.LT.10.)YLOGMN=1.
      IF(OMEGMN.LT.1.)YLOGMN=0.1
      IF(OMEGMN.LT.0.1)YLOGMN=0.01
      IF(OMEGMN.LT.0.01)YLOGMN=0.001
      IF(OMEGMX.GT.0.1)YLOGMX=0.1
      IF(OMEGMX.GT.1.0)YLOGMX=1.0
      IF(OMEGMX.GT.10.0)YLOGMX=10.0
      IF(OMEGMX.GT.100.0)YLOGMX=100.0
      IF(OMEGMX.GT.1000.0)YLOGMX=1000.0
      IF(OMEGMX.GT.1000.0)YLOGMX=10000.0
      DUMMY3=YLOGMX/YLOGMN
      NUMDEC=ALOG10(DUMMY3)
      IX=0
      DUMMY4=NUMDEC
      DUMMY5=LY
      YS=DUMMY4/DUMMY5
      IFINAL=LX+1
      JFINAL=LY+1
      DO 57 IA=1,IFINAL
      X(JA)=XS*(IA-1-IY)
      Y(JA)=YS*(JA-1)
   56 CALL DRAW(JA,X,Y,MOD,0,LA(50),ITITLE,XS,YS  ,0,IY,2,2,LX,LY,0,LAST)
   57 MOD=2
      DO 58 IB=1,NUMDEC
      DO 58 JB=1,10
      J=ALOG10(YLOGMN)*10.+10.+(IB-1)*10.+JB
      YLOG=JB
 1058 DO 1C58 KB=1,IFINAL
      X(KB)=ALOG10(YLOG)+IB-1
   58 CALL DRAW(KB,X,Y,MOD,0,LA(J),ITITLE,XS,YS  ,0,IY,2,2,LX,LY,0,LAST)
      IF((IPAR7).NE.(1))GO TO 532
   59 IF(IWRITE.LE.0)GO TO 200
C**********GRAPHICAL INFORMATION PRINTOUT SECTION***************
      WRITE(6,60) IRUN, NPARAM
   60 FORMAT(1H1,10X,'THE PRELIMINARY DATA FOR RUN NUMBER',I2,
     ε'  PARAMETER NUMBER',I2,' IS  *********.',//)
      IF(NGAMMA.GE.1) WRITE(6,61) ZGAMMA(IGAMMA)
   61 FORMAT(10X,'THE FOLLOWING CALCULATIONS WILL BE BASED ON THE THIRD
     εVARIABLE PARAMETER VALUE ,GAMMA=',1PE12.5,//)
      WRITE(6,62) (ITITLE(I),I=1,12)
   62 FORMAT(10X,'THE GRAPH WILL BE TITLED:',//,10X,6A8,//,10X,6A8,//)
      IF(NPARAM.NE.7) GO TO 80
      WRITE(6,83)
```

0325
0326
0327
0328
0329
0330
0331
0332
0333
0334
0335
0336
0337
0338
0339
0340
0341
0342
0343
0344
0345
0346
0347
0348
0349
0350
0351
0352
0353
0354
0355
0357
0358
0359
0360
0361
0362
0363
0364
0365
0366
0367
0368

```
      WRITE(6,63)DBMIN,DBMAX,XS                                         0369
   63 FORMAT(10X,'MAGNITUDE; THE DEPENDENT VARIABLE, WILL BE PLOTTED ON  0370
     &THE ORDINATE FROM',F10.4,' TO',F10.4,' AT',F10.4,' DB/DIVISION',/) 0371
      IF(IPHASE) 73,64,65                                               0372
   64 WRITE(6,65)                                                        0373
   65 FORMAT(10X,'PHASE CURVES WILL BE PLOTTED ON MAGNITUDE GRAPH',/)    0374
      GO TO 68                                                          0375
   66 WRITE(6,67)                                                        0376
   67 FORMAT(10X,'PHASE CURVES WILL BE PLOTTED ON SEPARATE GRAPH',/)     0377
   68 IF(IQUAD.GT.0) GO TO 70                                            0378
      WRITE(6,69)                                                       0379
   69 FORMAT(10X,'AUTOMATICALLY SCALED AT 45DEG/IN,-180 ON 0DB MAG.',/)  0380
      GO TO 73                                                          0382
   70 WRITE(6,71) IQUAD                                                  0383
   71 FORMAT(10X,'WITH A SCALE OF',I3,' DEG/IN, STARTING AT 0 DEG.',/)   0384
   72 WRITE(6,62) (JTITLE(I),I=1,12)                                     0385
   73 IF(IPCLAR.LE.0) GO TO 75                                           0386
      WRITE(6,74)XX                                                     0387
      XX                                                                0388
   74 FORMAT(10X,'A POLAR PLOT OF MAGNITUDE VS. PHASE WILL BE PLOTTED, W 0389
     &ITH MAGNITUDE SCALED AT',F10.5,' UNITS PER INCH.',/)              0390
      WRITE(6,62)(KTITLE(I),I=1,12)                                     0391
   75 IF(MODORD-2)127,121,121                                           0392
   76 FORMAT(10X,A8,' THE DEPENDENT VARIABLE, WILL BE PLOTTED ON THE     0393
     &ORDINATE FROM',1PE10.3,' TO',1PE10.3,' WITH A SCALE OF',1PE10.3,/) 0394
   77 FORMAT(10X,A8,' THE INDEPENDENT VARIABLE, WILL BE PLOTTED ON THE   0395
     &ABCISSA FROM',1PE10.3,' TO',1PE10.3,' WITH A SCALE OF',1PE10.3,/)  0396
   78 FORMAT(10X,A8,' WILL BE HELD CONSTANT AT THE VALUE OF',1PE12.5,/)  0398
   79 FORMAT(10X,A8,' WILL BE GRAPHED AS',I3,' CURVES CONSISTING OF TH   0399
     &E FOLLOWING VALUES:',/,16(20X,'CURVE',I3,' = ',1PE12.5,/))         0400
   80 IF(IMAG.EQ.0) WRITE(6,83)                                          0401
      IF(IMAG.EQ.1) WRITE(6,84)                                         0402
      IF(IMAG.EQ.2) WRITE(6,85)                                         0403
      IF(MODORD-2) 82,94,100                                            0404
   81                                                                   0405
   82 WRITE(6,76) JOKER(4),BETAMN,BETAMX,YS                             0406
   83 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS WILL BE IN DECIBELS',/)     0407
   84 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS ARE ACTUAL MAGNITUDE',/)    0408
   85 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS ARE MAGNITUDE SQUARED',/)   0409
      GO TO (86,86,121,130,121,200,200),ALPHMX,XS                       0410
   86 WRITE(6,77) JOKER(5),ALPHMN,ALPHMX,XS                             0411
      IF(NPARAM-1) 88,88,99                                            0412
   88 WRITE(6,78) JOKER(7),CONMAG                                        0413
   90 WRITE(6,79) NOMEGA,(I,ZOMEGA(I),I=1,NOMEGA)                        0414
   94 IF(NPARAM-5)200,125,200                                           0415
      WRITE(6,76) JOKER(5),BETAMN,BETAMX,YS                             0416
   96 GO TO (97,97,127,111,111,127),ALPHMN,ALPHMX,XS
   97 WRITE(6,77) JOKER(4),ALPHMN,ALPHMX,XS
      IF(NPARAM-1)88,88,99
   99 WRITE(6,78) JOKER(5),CONOME
```

163

```
0417  101 WRITE(6,79)JOKER(7),NMAG,(I,ZMAGSQ(I),I=1,NMAG)
0418  111 IF(NPARAM-4)200,139,200
0419  113 WRITE(6,78) JOKER(4),CONALP
0420  114 WRITE(6,114)
0421  115 FORMAT(10X,'THE COMBINED COEFFICIENTS OF THE NUMERATOR ARE',/)
0422      DO 116 I=1,LOE
0423      A(I)=HA(I)+HAT(I)*CONST
0424      B(I)=HBT(I)+HCT(I)*CONST
0425      KA=I-1
0426  116 WRITE(6,117) A(I),B(I),KA
0427  117 FORMAT(20X,1H(,E10.4,3H + , E10.4,16H * BETA ) * S**(,I2,1H)/)
0428      WRITE(6,136)
0429      DO 119 I=1,NOE
0430      D(I)=HD(I)+HDT(I)*CONST
0431      E(I)=HET(I)+HFT(I)*CONST
0432      KA=I-1
0433  119 WRITE(6,117) D(I),E(I),KA
0434  120 GO TO (200,200,200,101,90,200,139,200),NPARAM
0435  121 WRITE(6,79)JOKER(5),NBETA,(I,ZBETA(I),I=1,NBETA)
0436      IF(NPARAM-6)124,138,111
0437  124 WRITE(6,78) JCKER(6),CONOME
0438  125 WRITE(6,77) JCKER(7),XMAGMN,XMAGMX,XS
0439      GO TO 200
0440  127 WRITE(6,79)JOKER(4),NALPHA,(I,ZALPHA(I),I=1,NALPHA)
0441      IF(NPARAM-6)124,138,130
0442  130 WRITE(6,78) JOKER(5),CONBET
0443  131 FORMAT(10X,'BETA WILL BE HELD CONSTANT AT THE  VALUE OF',F10.4,/)
0444  132 WRITE(6,114)
0445      DO 134 I=1,LOE
0446      A(I)=HA(I)+HBT(I)*CONST
0447      B(I)=HAT(I)+HCT(I)*CONST
0448      KA=I-1
0449  134 WRITE(6,135)A(I),B(I),KA
0450  135 FORMAT(20X,1H(,E10.4,3H + ,E10.4,17H * ALPHA ) * S**(,I2,1H)/)
0451      WRITE(6,136)
0452  136 FORMAT(10X,'THE COMBINED COEFFICIENTS OF THE DENOMINATOR ARE-',/)
0453      DO 137 I=1,NCE
0454      D(I)=HD(I)+HET(I)*CONST
0455      E(I)=HDT(I)+HFT(I)*CONST
0456      KA=I-1
0457  137 WRITE(6,135) D(I),E(I),KA
0458  138 GO TO 120
0459  139 WRITE(6,78) JCKER(7),CONMAG
0460  140 WRITE(6,140)
0461      FORMAT(10X,'OMEGA,THE INDEPENDENT VARIABLE,WILL BE PLOTTED ON THE
0462     &ABCISSA.',/)
0463  141 IF(NPARAM-6)142,142,144
0464      IF(IGRAPH)142,142,144
```

```
142 WRITE(6,146) OMEGMN,OMEGMX,XS
    GO TO 200
143 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 142
144 WRITE(6,145)NUMDEC,YLOGMN
145 FORMAT(10X,'WITH A SPAN OF',I2,' DECADES, STARTING AT',F10.4,//30X
    &'*****NOTE*',//10X,'FOR THIS LOG PLOT, IT WILL BE NECESSARY
    &TO SHIFT THE ORIGIN TO THE LOWER RIGHT HAND CORNER',//10X,'WITH
    &THE ABCISSA RUNNING UP THE RIGHT HAND EDGE, AND THE ORDINATE INCRE
    &ASING TOWARDS THE LEFT ALONG THE BOTTOM',//)
146 FORMAT(10X,'FROM THE INITIAL VALUE OF',1PE10.3,'TO THE FINAL VALUE
    & OF',1PE10.3,' WITH A SCALE OF',1PE10.3,' RADIANS PER INCH.',/)
C************************************************************************
200 IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 203
    WRITE(6,201)IRUN,NPARAM,IWRITE
201 FORMAT(1H1,10X,'****THE OUTPUT DATA FOR RUN NUMBER',I2,',PAR
    &AMETER NUMBER',I2,' IS',///,10X,I2,' COMPUTATIONS WILL
    &BE PERFORMED BETWEEN EACH PRINTOUT.',///)
    IF(NGAMMA.GT.0) WRITE(6,202) ZGAMMA(IGAMMA)
202 FORMAT(10X,'THE VALUE OF GAMMA, THE THIRD VARIABLE IS:',1PE10.3,/)
203 DO 520 LCM=1,NLIMIT
    JCURVE=0
    DO 519 ICCEF=1,3,2
519 ZSIGN=ICOEF-2
    IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 208
    WRITE(6,205)LOM
205 FORMAT(//,10X,'COMPUTATIONAL RESULTS FOR CURVE NUMBER',I3)
    IF(NPARAM-7)206,208,208
206 WRITE(6,207)ZSIGN
207 FORMAT(1H+,55X,' USING A QUADRATIC FORMULA COEFFICIENT OF',F4.1,/)
208 IP=0
    YMAGPK=0.0
    OMEGPK=0.0
    DO 518 INCRMT=1,900
209 OMEGA=ZOMEGA(LOM)
    GO TO (209,216,253,216,212,254,257,1000),NPARAM
211 CONST=ALPHMN+(INCRMT-1)*(ALPHMX-ALPHMN)/899.0
212 IF(MODORD-2)260,213,213
213 DO 214 I=1,LGE
214 A(I)=HA(I)+HAT(I)*CONST
    B(I)=HBT(I)+HCT(I)*CONST
    DO 215 I=1,MCE
215 C(I)=HD(I)+HDT(I)*CONST
    E(I)=HET(I)+HFT(I)*CONST
    GO TO 263
216 IF(IMAG-1)217,218,219
217 DUMMY1=10.0**(ZMAGSQ(LOM))/10.0)
    GO TO 220
218 DUMMY1=ZMAGSQ(LCM)*ZMAGSQ(LCM)
```

```
0524          GO TO 220
0525  219     DUMMY1=ZMAGSQ(LCM)
0526  220     IF(NPARAM-2)211,211,212
0527  2203    OMEGA=CONOME
0528  2254    IF(MODORD-2)255,256,1000
0529  255     CONST=ZBETA(LOM)
0530          GO TO 260
0531  256     CONST=ZALPHA(LOM)
0532          GO TO 213
0533  257     IF(MODORD-2)259,258,258
0534  258     BETA=ZBETA(LOM)
0535          CONST=CCNALP
0536          GO TO 213
0537  259     BETA=ZALPHA(LCM)
0538          CONST=CCNBET
0539  260     DO 261 I=1,LCE
0540  261     A(I)=HA(I)+HBT(I)*CONST
0541          B(I)=HAT(I)+HCT(I)*CONST
0542          DO 262 I=1,MOE
0543          D(I)=HD(I)+HET(I)*CONST
0544  262     E(I)=HDT(I)+HFT(I)*CONST
0545  263     GO TO (350,350,275, 290,264 ,291,294,1000),NPARAM
0546  264     OMEGA=ZOMEGA(LOM)
0547  275     ZMAG=XMAGMN+(INCRMT-1)*(XMAGMX-XMAGMN)/899.0
0548          IF(IMAG-1)276,277,278
0549  276     DUMMY1=10.0**(ZMAG/10.0)
0550          GO TO 350
0551  277     DUMMY1=ZMAG*ZMAG
0552          GO TO 350
0553  278     DUMMY1=ZMAG
0554          GO TO 350
0555  290     IF(IGRAPH)292,292,294
0556  291     IF((IGRAPH.GE.0).AND.(IGRAPH.LE.1)) GO TO 294
0557  292     OMEGA=OMEGMN+INCRMT*(OMEGMX-OMEGMN)/900.0
0558          GO TO 350
0559  294     WINC=1.0+(INCRMT-1)*NUMDEC/899.0
0560          OMEGA=(YLOGMN*10.0**WINC)/10.0
0561  C*******NUMERATCR COEFFICIENT CALCULATIONS*****************
0562  350     SNB2=0.0
0563          SNB1=0.0
0564          SNB0=0.0
0565          SUMN=0.0
0566          NDOUN=2*NORN
0567          DO 373 ILOOP=1,4
0568          DO 373 NP=2,LOE
0569          DO 373 MQ=1,NORN
0570          MP=MQ-1
0571          NQ=NP-1
```

```
361   IF(((NP+MQ).GT.NDOUN).AND.(NORN.NE.1)) GO TO 373
362   IF((MP+NQ)-NORN)362,362,373
363   IF(NP-MP)373,373,363
364   IF(MP)364,364,366
      C=1.0
      GO TO 367
366   C=2.0
367   GO TO (368,369,370,371),ILOOP
368   CSNB2 = (C*B(NP+MP)*B(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SNB2 = SNB2 + CSNB2
      GO TO 373
369   CSNB1 =(C*A(NP+MP)*B(NP-MP)+C*A(NP-MP)*B(NP+MP))*OMEGA**(2*NQ)*(-
     11.0)**(MQ+1)
      SNB1 = SNB1 + CSNB1
      GO TO 373
370   CSNB0 = (C*A(NP+MP)*A(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SNB0 = SNB0 + CSNB0
      GO TO 373
371   COEFN =(C*A(NP+MP)*A(NP-MP)+C*A(NP-MP)*B(NP+MP)*BETA+C*A(NP-MP)*B
     1(NP+MP)*BETA+B(1)*B(1)*BETA*BETA
     2(MQ+1))*BETA*C*B(NP+MP)*BETA*BETA)*OMEGA**(2*NQ)*(-1.0)**
      SUMN = SUMN + COEFN
373   CONTINUE
      SNB2 = SNB2 + B(1)*B(1)
      SNB1 = SNB1 + 2.0*A(1)*B(1)
      SNB0=SNB0+A(1)*A(1)
      SUMN = SUMN + A(1)*A(1)+2.0*A(1)*B(1)*BETA+B(1)*B(1)*BETA*BETA
C*****DENOMINATOR COEFFICIENT CALCULATIONS********************
      SDB2=0.0
      SDB1=0.
      SDB0=0.
      SUMD=0.
      NDOUD=2*NORD
      DO 385 ILCCP=1,4
      DO 385 NP=2,MOE
      DO 385 MQ=1,NORD
      MP=MQ-1
      NQ=NP-1
375   IF(((NP+MQ).GT.NDOUD).AND.(NORD.NE.1)) GO TO 385
376   IF((MP+NQ)-NORD)376,376,385
377   IF(NP-MP)378,378,379
378   C=1.0
      GO TO 380
379   GO TO (381,382,383,384),ILOOP
380   C=2.0
381   CSDB2 = C*E(NP+MP)*E(NP-MP)*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SDB2 = SDB2 + CSDB2
```

167

```
      GO TO 385
382 CSDB1 =(C*D(NP+MP)*E(NP-MP)+C(NP-MP)*E(NP+MP))*OMEGA**(2*NQ)*(-
   11.0)**(MQ+1)
      SDB1 = SDB1 + CSDB1
      GO TO 385
383 CSDB0 =(C*D(NP+MP)*D(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SDB0 = SDB0 + CSDB0
      GO TO 385
384 COEFD =(C*D(NP+MP)*D(NP-MP)+C*D(NP-MP)*E(NP+MP))*E(NP-MP)*E
   1(NP+MP)*BETA+C*E(NP-MP)*E(NP+MP)*BETA*BETA)*OMEGA**(2*NQ)*(-1.0)**
   2(MQ+1)
      SUMD = SUMD + CCEFD
385 CONTINUE
      SUMD = SUMD + D(1)*D(1)+2.0*D(1)*E(1)*BETA+E(1)*E(1)*BETA*BETA
      SDB2 = SDB2 + E(1)*E(1)
      SDB1 = SDB1 + 2.0*D(1)*E(1)
      SDB0 = SDB0 + D(1)*D(1)
      IF(NPARAM-7)413,386,1000
C*******************PARAM 7 PHASE COMPUTATIONAL SECTION**************
386 TOE=LOE/2.0
      PNEVEN=0.
      PDEVEN=0.
      PNODD=0.
      PDODD=0.
      IF(NCRN.EQ.0) GO TO 393
      JOE=LOE/2
      IF(JOE-TOE)387,388,388
387 JON=(LOE+1)/2
      MON=JON
      GO TO 389
388 JON=(LOE+2)/2
      MON=JON-1
      IF(NCRN-1)393,389,389
389 DO 390 I=2,JON
390 PNODD=PNODD+(A(2*I-2)+B(2*I-2)*BETA)*OMEGA**(2*I-3)*(-1.0)**(I)
      IF(NORN-1)393,393,391
391 DO 392 I=2,MON
392 PNEVEN=PNEVEN+(A(2*I-1)+B(2*I-1)*BETA)*OMEGA**(2*I-2)*(-1.)**(I-1)
393 PNEVEN=PNEVEN+A(1)+B(1)*BETA
      TOE=MOE/2.0
      JOE=MOE/2
      IF(JOE-TOE)394,395,395
394 JON=(MOE+1)/2
      MON=JON
      GO TO 396
395 JON=(MOE+2)/2
      MON=JON-1
      IF(NORD-1)400,396,396
```

```
396     DO 397 I=2,JCN
397     PDODD=PDODD+(D(2*I-2)+E(2*I-2)*BETA)*OMEGA**(2*I-3)*(-1.0)**(I)        0667
        IF(NORD-1)400,400,398                                                   0668
398     DO 399 I=2,MOR                                                          0669
399     PDEVEN=PDEVEN+(D(2*I-1)+E(2*I-1)*BETA)*OMEGA***(2*I-2)*(-1.)**(I-1)     0670
400     PDEVEN=PDEVEN+D(1)+E(1)*BETA                                            0671
        TANGIM=PDEVEN*PNODDC-PNEVEN*PDODD                                       0672
        TANGRE=PNEVEN*PDEVEN-PNODD*PDODD                                        0673
        TANG=TANGIM/TANGRE                                                      0674
        RAD=ATAN(TANG)                                                          0675
        IF((TANGRE.LT.0.0).AND.(TANGIM.LT.0.0)) JQUAD=1                         0676
        IF((TANGRE.LT.0.0).AND.(TANGIM.GT.0.0)) JQUAD=-1                        0677A
        IF((TANGRE.GT.0.0).AND.(TANGIM.LT.0.0)) RAD=JQUAD*(3.141593+JQUAD*RAD)  0677B
        IF((TANGRE.GT.0.0).AND.(TANGIM.LT.0.0)) JQUAD=-1
        IF((TANGRE.GT.0.0).AND.(TANGIM.GT.0.0)) JQUAD=1
402     YMAGSQ=ABS(SUMN/SUMD)                                                   0678
        YMAG=SQRT(YMAGSQ)                                                       0679
        YMAGDB=20.0*.434294S*ALOG(YMAG)                                         0680
        YMAGAB=ABS(YMAGDB)                                                      0681
        IF(YMAGPK-YMAGDB)403,404,404                                           0682
403     YMAGPK=YMAGDB                                                           0683
        OMEGPK=OMEGA                                                            0684
        IF(DBMAX-YMAGDB)405,406,406                                            0685
404     YMAGDB=DBMAX                                                            0686
405     IF(YMAGDB-DBMIN)407,408,408                                            0687
406     YMAGDB=DBMIN                                                            0688
407     IF(XS*LX )-YMAGAB)409,410,410                                          0689
408     YMAGAB=LX-XS                                                            0690
409     GO TO 426                                                              0691
410   **************QUADRATIC FORMULA SOLN. OF DEPENDENT VARIABLE, PARAM1-6     0692
C***
413     ACO=DUMMY1*SDB2-SNB2                                                    0693
416     BCO=DUMMY1*SDB1-SNB1                                                    0694
        CCO=DUMMY1*SDB0-SNB0                                                    0695
        IF(ABS(ACO).LT.1.0E-35) GO TO 425                                       0696
        DPP=BCO*BCO-4.0*ACO*CCO                                                 0697
        IF(DPP.LT.0.0)GO TO 425                                                 0698
        DSQR=ZSIGN*SQRT(DPP)                                                    0699
        BETA=(DSQR-BCO)/(2.0*ACO)                                               0700
        IF(BETA-BETAMN)425,426,424                                             0701
424     IF(BETA-BETAMX)426,426,425                                            0702
425     IF(IP-1)500,505,507                                                    0703
426     IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 450                        0704
C***  **************COMPUTED DATA PRINTOUT SECTION***********************       0705
        ISTEP=ISTEP+1                                                           0706
        IF(IP)428,428,427                                                       0707
427     IF(ISTEP-IWRITE)450,430,430                                            0708
428     WRITE(6,429)LCM                                                        0709
429     FORMAT(/,10X,'        ALPHA        BETA        MAGNITUDE        OMEGA    0710
```

```
      & TANGENT   ;  CURVE NUMBER',I2,/)
  430 IF(MODORD.EQ.2) GO TO 431
      GO TO (433,435,436,437,439,441,443,1000),NPARAM
  431 GO TO (432,434,445,438,440,442,444,1000),NPARAM
  432 WRITE(6,6)   CONST,BETA,CONMAG,CMEGA
      GO TO 448
  433 WRITE(6,6)     BETA,CONST,CONMAG,CMEGA
      GO TO 448
  434 WRITE(6,6)   CONST,BETA,ZMAGSQ(LCM),OMEGA
      GO TO 448
  435 WRITE(6,6)     BETA,CONST,ZMAGSQ(LCM),OMEGA
      GO TO 448
  436 WRITE(6,6)     BETA,CONST,ZMAG,CMEGA
      GO TO 448
  437 WRITE(6,6)     BETA,CONBET,ZMAGSQ(LCM),OMEGA
      GO TO 448
  438 WRITE(6,6)   CCNALP,BETA,ZMAGSQ(LCM),CMEGA
      GO TO 448
  439 WRITE(6,6)     BETA,CONBET,ZMAG,CMEGA
      GO TO 448
  440 WRITE(6,6)   CCNALP,BETA,ZMAG,CMEGA
      GO TO 448
  441 WRITE(6,6)     BETA,CCNST,CONMAG,CMEGA
      GO TO 448
  442 WRITE(6,6)   CONST,BETA,CONMAG,CMEGA
      GO TO 448
  443 WRITE(6,6)   ZALPHA(LCM),CONBET,YMAGCB,OMEGA,TANG
      GO TO 448
  444 WRITE(6,6)   CCNALP,ZBETA(LCM),YMAGDB,OMEGA,TANG
      GO TO 448
  445 WRITE(6,6)   CONST,BETA,ZMAG,CMEGA
  448 ISTEP=0
C*********************GRAPHICAL ARRAY STCRAGE SECTION**********************
  450 IP=IP+1
      Y(IP)= BETA
      GO TO (453,453,460,470,460,470,475 ,1000),NPARAM
  453 X(IP)= CONST
      GO TO 500
  460 X(IP)=ZMAG
      GO TO 500
  470 X(IP)=OMEGA
      IF(NPARAM-6)471,472,475
  471 IF(IGRAPH)500,500,474
  472 IF(IGRAPH.LT.0).OR.(IGRAPH.GT.1) GO TO 500
  474 X(IP)=LX-BETA
      GO TO 479
  475 X(IP)=-YMAGDB
      IC(LCM)=IP
```

```
      IF(IPHASE.GE.0).OR.(NGAMMA.LT.0)) GO TO 4475
      S(LOM,IP)=(ABS(YMAG)*COS(RAD))/XX
      T(LOM,IP)=(ABS(YMAG)*SIN(RAD))/XX
      GO TO 479
 4475 V(LOM,IP)=(ALOG10(OMEGA/YLOGMN)*LY/NUMDEC)*YS
      IF(NGAMMA.LT.0) GO TO 4477
      IF(IQUAD.LE.0) GO TO 476
      IF(RAD.LE.0.0) W(LOM,IP)=JQUAD*(IY-JQUAD*RAC*57.29256/IQUAD)*XS
      IF(RAD.GT.0.0) IZ=8-IY
      IF(RAD.GT.0.0) W(LOM,IP)=JQUAD*(IZ-JQUAD*RAC*57.29256/IQUAD)*XS
      GO TO 477
 476  W(LOM,IP)=JQUAD*(3.141593-JQUAC*RAD)/0.7853981)*XS
 477  IF(W(LOM,IP).GT.(-DBMIN)) W(LOM,IP)=-DBMIN
      IF(W(LOM,IP).LT.(-DBMAX)) W(LOM,IP)=-DBMAX
      GO TO 479
 4477 U(LOM,IP)=U(LCM,IP)+YMAG
 479  Y(IP)=(ALOG10(OMEGA/YLOGMN)*LY/NUMDEC)*YS
 500  IF(INCRMT.LT.900) GO TO 518
C*************************GRAPHICAL CURVE PLOTTING**********************
 501  IF(IP-1)501,505,507
      IF(JCURVE.GT.0) GO TO 503
 502  FORMAT(10X,'NO GRAPH POINTS WERE FOUND FOR CURVE NUMBER ',I2)
      IF(NPARAM.NE.7) WRITE(6,207) ZSIGN
      GO TO 518
 503  WRITE(6,504)LOM
 504  FORMAT(10X,'NC FURTHER POINTS WERE FCUND FOR CURVE NO.',I3) ZSIGN
      IF(NPARAM.NE.7) WRITE(6,207) ZSIGN
      GO TO 518
 505  WRITE(6,506)
 506  FORMAT(10X,'ONLY ONE POINT WAS FCUND, AND WILL NOT BE PLOTTED',/)
      GO TO 516
 507  J=LOM+50
      IF(ZSIGN.GT.0.0).AND.(JCURVE.GT.0))J=50
      JCURVE=1
      IF(IGRAPH.LE.-2) GO TO 516
      IF(NPARAM.EQ.7).AND.(IPAR7.LT.NPAR7))GO TO 518
      WRITE(6,511)LCM
 511  FORMAT(/,10X,'THE PREVIOUS POINTS WILL BE PLOTTED AS CURVE ',I2,/)
      IF(NPARAM-7)514,512,514
 512  WRITE(6,513) YMAGPK,CMEGPK
 513  FORMAT(/,10X,'THE MAXIMUM VALUE OF MAGNITUDE IS',F10.4,' LOCATED A
     &T OMEGA EQUAL TO',F10.4,/)
      J=50
 514  CALL DRAW(IP,X,Y,MOD,0,LA(J),ITITLE,XS,YS,IX,IY,2,2,LX,LY,1,LAST)
      WRITE(6,8888) LAST
 8888 FORMAT (//,7X,'LAST =',I3)
      MOD=2
```

171

```
516  IP=0
518  CONTINUE
     IF(NPARAM.EQ.7) GO TO 520
519  CONTINUE
520  CONTINUE
C*****GAMMA & PARAM-7 SUPERPOSITION. SEE INPUT DATA SYMBOL NGAMMA****
517  IF((IPAR7.GE.NPAR7).OR.(NPARAM.NE.7)) GO TO 529
     IF(IPAR7.GT.1)GO TO 523
     IF((NGAMMA.LE.0).OR.(IGAMMA.GT.NGAMMA)) GAMMA=0.0
     DO 521 I=1,LOE
     OLDHA(I)=HA(I)
     OLDHAT(I)=HAT(I)
     OLDHBT(I)=HBT(I)
     OLDHCT(I)=HCT(I)
521  OLDHD (I)=HD(I)
     DO 522 I=1,MOE
     OLDHD (I)=HD(I)
     OLDHDT(I)=HDT(I)
     OLDHET(I)=HET(I)
522  OLDHFT(I)=HFT(I)
523  IPAR7=IPAR7+1
     IF(NGAMMA.GT.0) GO TO 526
     READ(5,5) (HA(I), I=1,LOE)
     READ(5,5) (HAT(I), I=1,LOE)
     READ(5,5) (HBT(I), I=1,LOE)
     READ(5,5) (HCT(I), I=1,LOE)
     READ(5,5) (HDT(I), I=1,MOE)
     READ(5,5) (HET(I), I=1,MOE)
     READ(5,5) (HFT(I), I=1,MOE)
     IF(IPAR7.LT.NPAR7) GO TO 200
     DO 525 LOM=1,NLIMIT
     IP=IC(LOM)
     DO 524 I=1,IP
     X(I)=-U(LCM,I)
524  Y(I)=V(LOM,I)
     J=LOM+1
525  CALL DRAW(IP,X,Y,MOD,0,LA(J),ITITLE,XS,YS,IX,IY,2,2,LX,LY,1,LAST)
526  DO 527 I=1,LOE
     HA(I)=OLDHA(I)+GAMMA*GAMHA(I)
     HAT(I)=OLDHAT(I)+GAMMA*GAMHAT(I)
     HBT(I)=OLDHBT(I)+GAMMA*GAMHBT(I)
527  HCT(I)=OLDHCT(I)+GAMMA*GAMHCT(I)
     DO 528 I=1,MOE
     HD(I)=OLDHD (I)+GAMMA*GAMHD (I)
     HDT(I)=OLDHDT(I)+GAMMA*GAMHDT(I)
     HET(I)=OLDHET(I)+GAMMA*GAMHET(I)
528  HFT(I)=OLDHFT(I)+GAMMA*GAMHFT(I)
     IF(IGAMMA.LE.NGAMMA) GO TO 26
```

0802
0803
0804
0805
0806
0807
0808
0809
0810
0811
0812
0813
0814
0815
0816
0817
0818
0819
0820
0821
0822
0823
0824
0825
0826
0827
0828
0829
0830
0831
0832
0833
0834
0835
0836
0837
0838
0839
0840
0841
0842
0843
0844
0845
0846
0847
0848
0849

```
      529 IF((NPARAM.NE.7).OR.(IGRAPH.LE.-2)) GO TO 999                   0850
C**************************PARAM 7 PHASE CURVES***********************     0851
          IPAR7=2                                                         0854
          IF(IPHASE)537,532,999                                           0855
      532 DO 536 LOM=1,NLIMIT                                             0856
          IP=IC(LOM)                                                      0857
          DO 533 I=1,IP                                                   0858
          X(I)=W(LOM,I)                                                   0859
      533 Y(I)=V(LOM,I)                                                   0860
      536 CALL DRAW(IP,X,Y,MOD,0,LA(50),JTITLE,XS,1.,0,IY,2,2,LX,LY,1,LAST) 0861
          WRITE(6,8888) LAST
      537 IPAR7=1
          GO TO 999
C***************************PARAM 7 POLAR PLOT CURVES*********************  0864
      777 MOD=1
      9999 DO 9999 I=1,12                                                 0866
          ITITLE(I)=KTITLE(I)                                             0867
          DO 542 I=1,8                                                    0868
          RADIUS=I/2.0                                                    0869
          DO 539 J=1,360                                                  0870
          RADIAN=6.28318/360*J                                           0871
          X(J)=RADIUS*COS(RADIAN)                                         0872
      539 Y(J)=RADIUS*SIN(RADIAN)                                         0873
          CALL DRAW(J,X,Y,MOD,0,LA(1),KTITLE,1.,1.,4,2,2,8,8,0,LAST)      0874
      542 MOD=2                                                           0875
          DO 543 J=10,180,10                                              0876
          X(1)=X(J)                                                       0877
          X(2)=-X(J)                                                      0878
          Y(1)=Y(J)                                                       0879
      543 Y(2)=-Y(J)                                                      0880
          CALL DRAW(2,X,Y,2,0,LA(1),KTITLE,1.,1.,4,2,2,8,8,0,LAST)        0881
          DO 545 LOM=1,NLIMIT                                             0882
          IP=IC(LCM)                                                      0883
          DO 544 J=1,IP                                                   0884
          X(J)=S(LCM,J)                                                   0885
      544 Y(J)=T(LCM,J)                                                   0886
          L=LOM+1
      545 CALL DRAW(IP,X,Y,2,0,LA(L),KTITLE,1.,1.,4,2,2,8,8,0,LAST)       0887
          WRITE(6,8888) LAST                                             0887A
          IPOLAR=0                                                        0888
      999 IF((MOD).NE.(2)) GO TO 1000                                     0889
C*************TERMINATING GRAPH LINE FOR EACH PLOT*********************    0890
          X(1) = 0.0                                                      0891
          Y(1) = 0.0                                                      0892
          X(2) = (LX-IY)*XS
          Y(2) = 0.0
          CALL DRAW(2,X,Y,3,0,LA(50),ITITLE,1.0,1.0,0,0,2,2,LX,LY,1,LAST)
          WRITE(6,8888) LAST
```

173

```
      MOD=1
      IF((IPAR7).EQ.(1)) GO TO 888
530   DO 531 I=1,12
531   ITITLE(I)=JTITLE(I)
      GO TO 55
888   IF((IPOLAR.EQ.1).AND.(NPARAM.EQ.7)) GO TO 777
1000  CONTINUE
      STOP
      END
//LINK.SYSLMOD DD SPACE=(CYL,(5,1,1))
//GO.SYSIN DD *
1       16      1       1       0       10
2       5       -1      0       1       4.
0       50              16      6       0.

1.      1.      -10.    50.     4.5   .01   1       10.
16.     16.     6.      5.      .5    6
3.5     8.      1.5     1.

1.
1.
1.
1764.   1260.   225.

1764.   1260.   1975.   537.5   42.5   1.

1750.   537.5   42.5    1.
```

                                                                    0893
                                                                    0894A
                                                                    0894B
                                                                    0894C
                                                                    0895
                                                                    0896
                                                                    0897
                                                                    0899

```
DAVIS,J.P.     CLOSED LOOP
EXAMPLE #7     MAG PLCT
DAVIS,J.P.     CLOSED LOOP
EXAMPLE #7     PHASE PLOT
```

LIST OF REFERENCES

1. Del Toro, V. and Parker, S.R., _Principles of Control Engineering_, McGraw-Hill Book Company, 1960.

2. Electronic Associates, Inc., _TR-20 Computer Operator's Reference Handbook_, 1964.

3. Glavis, G.O., _Frequency Response in the Parameter Plane_, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1968.

4. Ruivo, F.M.P., _Frequency Response Study of Nonlinear Networks by Parameter Plane Techniques_, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1968.

5. Staples, D.S., _Frequency Response Analysis of Nonlinear Dynamic Systems_, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1968.

6. Thaler, G.J. and Pastel, M.P., _Analysis and Design of Nonlinear Feedback Control Systems_, McGraw-Hill Book Company, 1962.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Commander Naval Ordnance Systems Command Hqs. Department of the Navy Washington, D.C. 20360	1
4.	Professor G. J. Thaler Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	10
5.	Dr. D. D. Siljak Department of Electrical Engineering University of Santa Clara Santa Clara, California 93153	1
6.	Mr. J. Karmarkar Department of Electrical Engineering University of Santa Clara Santa Clara, California 93153	1
7.	Dr. A. G. Thompson Department of Mechanical Engineering University of Adelaide Adelaide, SOUTH AUSTRALIA	1
8.	Dr. J. Peracaula Division of Automatic Control Technical University J. Gutierrez Abascal 2 Madrid, 6, SPAIN	1
9.	Dr. M. Rakic Marsala Tita 6 Belgrade, YUGOSLAVIA	1
10.	Capt. R. D. Staples, USMC 1816 Ridge Road El Dorado, Kansas 67042	1

11. Lt. F. M. P. Ruivo   1
     Ministerio da Marina
     Portuguese Navy
     Lisbon, PORTUGAL

12. Dr. Lawrence Eisenberg   1
     Department of Electrical Engineering
     Moore School of Engineering
     University of Pennsylvania
     Philadelphia, Pennsylvania 19104

13. Lt.JG John P. Davis, USN   1
     173 Little Neck Road
     Centerport, New York 11721

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Naval Postgraduate School Monterey, California 93940	Unclassified
	2b. GROUP

3. REPORT TITLE

Frequency Response Analysis and Design of Single-Valued Nonlinear
Systems Using the Parameter Plane

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis;   June 1969

5. AUTHOR(S) (First name, middle initial, last name)

John P. DAVIS

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1969	179	6

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Frequency response techniques are a valuable tool in the analysis
and synthesis of linear systems. Extension of these techniques is made
to analyze and design systems with a single-valued nonlinear element.
The relationship between the characteristics of a nonlinear element and
the frequency of the system is developed by simple calculations and a
digital computer program.

DD FORM 1 NOV 65 **1473**   (PAGE 1)

S/N 0101-807-6811

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Single-Valued Nonlinear Systems						
Parameter Plane						
Frequency Response Analysis						

DD FORM 1473 (BACK)
1 NOV 65

S/N 0101-807-6821

Unclassified
Security Classification

A-31409

180